Abstract

This thesis mainly focuses on the mathematical modeling and analysis of the coupled phenomena of uid ow and solid phase deformation (poroelastohydrodynamics) inside soft biomaterials, such as a tumor. In the recent past, one of the most studied topics is uid ow through biological tissues such as tumors, articular cartilage, Glycocalyx layers, and arterial tissue. Though the internal structure of biological tissues such as tumors is complicated, developing mathematical models at least as approximations are very useful. However, over the last 50 years, of 1.5 million papers devoted to the area of cancer research, only about 5% are concerned with mathematical modeling. Robust mathematical models and techniques can provide a qualitative description of the system dynamics and increase insight into the mechanisms that control tumor evolution and growth. Theoretical predictions generated from the mathematical models may also reduce the number of animal experiments that need to be carried out, suggest new experimental programmes, and identify optimal tumor therapy schedules. Hence, a considerable contribution could be made by Mathematics owing to the availability of experimental data. A systematic mathematical analysis is required for this. Both poroelasticity theory and mixture theory can be used to model such phenomena (as can other modeling approaches). Here, we use the biphasic mixture theory to develop mass and momentum balance equations to describe the coupled phenomena of uid ow and solid phase deformation in the tumor and the normal tissue regions. The mathematical models are governed by systems of linear and nonlinear partial di_erential equations (PDEs). These PDEs may be of the types: elliptic, parabolic, and hyperbolic. The systems of linear and nonlinear PDEs in the generic form, which is the outcome of the mathematical modeling in this thesis, are very complex and cover a large class of physical problems. It is very challenging to handle such a system of nonlinear-coupled PDEs either numerically or analytically. Thus, we simplify the generic mathematical models in the context of poroelastohydrodynamics under some biologically relevant assumptions, mainly (i) we assume only linear/small deformation in the system (ii) neither the tumor tissue nor the healthy tissue is growing so that the volume fractions are constants (iii) the interface between tumor and healthy tissue is _xed and smooth. These assumptions yield many interesting sub-problems, which are comparatively easy to understand from the mathematical and physical points of view. In general, it is very di_cult or impossible to _nd exact or closed-form solutions of such linear and nonlinear PDEs. Hence, the PDEs have been solved either by some numerical method or by analytical approximations. However, before tackling any system of PDEs numerically, one of the fundamental tasks is to establish the well-posedness (existence, uniqueness, and continuous dependence on data). The existence of a solution means a given model is coherent. At the same time, uniqueness and stability enhance the possibility of providing accurate numerical approximations. Hence, this thesis's signi_cant ontribution is to develop mathematical models that describe the coupled phenomena of uid ow and solid phase deformation inside soft biomaterials in particular tumors. Further, we establish the well-posedness of the governing models in a weak sense. Moreover, we develop 1D closed-form solutions in case of the spherically symmetric domain and analyze the behavior of uid ow, solid-phase deformation, and stress within a tumor.

Keywords: Deformable porous medium; In-vitro and in-vivo tumors; Biphasic mixture theory; Well-posedness; Sobolev spaces; Weak/variational formulation; Energy estimates; Lax-Milgram Lemma; Galerkin Method; Weak convergence; 1D spherical solution.