Abstract

In this thesis, we consider the projection methods to approximate the solutions of the linear and nonlinear weakly singular Volterra integral equations of the second kind using piecewise polynomials based on graded mesh and Jacobi polynomials. Generally, the exact solutions of the weakly singular Volterra integral equations is nonsmooth at the initial point of integration. First we develop the Galerkin and iterated Galerkin methods to obtain the convergence results in uniform norm for linear weakly singular Volterra integral equation by piecewise polynomials based on graded mesh. We also discuss the multi Galerkin and iterated multi Galerkin methods to obtain the superconvergence results in uniform norm. We extend these methods to nonlinear weakly singular Volterra integral equations of Hammerstein type and obtain the superconvergence results in uniform norm. However, in all these piecewise polynomially based projection methods to get more accurate approximate solutions, the norm of the partitions must be tend to zero, which leads indirectly to a large system of linear/nonlinear equations, which makes numerical computation very expensive. To reduce the computational complexity, we use the Jacobi polynomials as the basis for the approximation subspace $\mathbb{X}_N$. We discuss the Jacobi spectral Galerkin and iterated Jacobi spectral Galerkin methods for linear and nonlinear weakly singular Volterra integral equations of both Hammerstein and Urysohn types and obtain the convergence results in both the uniform and weighted L^2 norms. To obtain the superconvergence results, we transform the domain of integration of Volterra integral equations to the standard interval $[-1, 1]$ by using variable and function transformations. We then discuss the convergence analysis in two cases when the solution is sufficiently smooth and nonsmooth. We prove that the iterated Jacobi spectral Galerkin method improves over Jacobi spectral Galerkin method. We also discuss the Jacobi spectral multi Galerkin method and its iterated version, and we prove that the convergence results obtained in iterated Jacobi spectral multi Galerkin method improves over the iterated Jacobi spectral Galerkin method in both uniform and weighted L^2 norms under the same assumption as in the Jacobi spectral Galerkin method. The size of the system to be solved in Jacobi spectral multi Galerkin method remains same as in the Jacobi spectral Galerkin method. We illustrate these results by numerical examples.

Keywords: Volterra integral equations, Urysohn integral equations, Hammerstein integral equations, weakly singular kernel, Piecewise polynomials, Jacobi polynomials, Superconvergence results.