Chapter I

Introduction

1. The Field Equations in Material Media

Matter in Maxwell's theory was but a field of multipole moments while electricity itself was regarded as a continuous fluid. It is remarkable that the discoveries of the present century that did so much violence to the traditional picture of matter and robbed it of most of its substance should leave Maxwell's equations untouched. However, the new insight gained into the interior of matter opened up the possibility of a theoretical evaluation of the empirical permeabilities of Maxwell. This in turn demanded an understanding of the macroscopic formalism in terms of the microscopic concepts.

The first step on this path was taken by Lorentz[1] (1916) when he assumed the vacuum field equations to serve just as well in the interstitial spaces of the swarm of tiny charges and showed that the electrodynamics of matter follows as a consequence. Lorentz regarded the macroscopic field vectors as space averages of the microscopic ones over volume elements that were supposed to be physically small and yet contain a large number of charges. Such an averaging is an operation that can be interchanged with space and time differentiation and validates two of Maxwell's equations trivially, while the other two follow from a multipole expansion of the microscopic source densities. With minor modifications Lorentz's approach was adopted by several authors in formulating their theories of susceptibility [Van Vleck[2] (1932)].

Nevertheless, the loosely defined volume elements of Lorentz remained a lacuna in his theory, for, in many practical situations no such small volume elements exist which contain a large enough number of charges to lead to an adequately continuous definition of the quantities involved [Robinson[3] (1971)]. Besides, the polarization and magnetization vectors that are here regarded as space averages are often defined as ensemble averages in their actual evaluation from molecular theories [Kirkwood[4] (1936)]. Increased sophistication in the latter naturally called for a corresponding refinement in the original ideas of Lorentz. Thus the connection of the macroscopic field equations to the microscopic ones has continued to attract attention and is often termed as the 'derivation' of Maxwell's equations. Further, the vexing question of the field energy-momentum tensor in polarized media remains unresolved as long as macroscopic arguments alone are resorted to [Abraham[5] (1909), Minkowski[6] (1910)]. The problem however becomes well-defined if one starts with the microscopic concepts and considers the total (field plus matter) energy-momentum conservation laws [Lorentz[7] (1904), Einstein and Laub[8] (1908), de Groot and Suttorp[9] (1967, 1968)].

The need to postulate physically small volume elements was eliminated by Mazur and Nijboer[10] (1953) by employing a different technique based on ensemble averaging. The averaging here proceeds in the phase space of all the charged particles making up the medium, with a very general distribution function as the weight factor. Conservation of probability density ensures

that differentiation with respect to time can be interchanged with integration in phase space. An expansion of the atomic distribution function in a series over the internal coordinates leads right away to the equations of electrodynamics. The space averaging of Lorentz can in fact be looked upon as a peculiar kind of ensemble averaging with a very special distribution function such that all the configurations taken into account are connected to one another by a rigid displacement of the system as a whole.

A calculation of the permeabilities often involves quantum mechanical considerations even though Maxwell's equations themselves describe no quantum effect. Starting from a quantum analogue of the Lorentz field equations and a suitably defined density operator, Schram[11] (1960) gave a purely quantum statistical derivation of Maxwell's equations. The macroscopic field variables turn out once again to be the ensemble averages of the microscopic field operators connected with the system of charged particles interacting with a quantized electromagnetic field. Space and time differentiation commute with the operation of taking trace and the matrix elements of the density operator can be expanded in powers of atomic variables in a coordinate representation. As the diagonal elements of the density matrix are the quantum analogues of the classical distribution function, Schram's treatment bears a strong resemblence to that of Mazur and Nijboer.

Higher order multipole moments are customarily ignored since their effects are weak and hard to detect experimentally.

The availability of intense sources of coherent illumination has changed the picture and permitted the detection of quadrupolar effects in situations where they would ordinarily be swamped by the dipole effects. As Pershan[12] (1963) has pointed out, an induced quadrupole moment can sometimes be solely responsible for the generation of a second harmonic component, e.g. in an isotropic medium of centro-symmetric molecules. Retention of higher order moments is also vital to the molecular theories of the equation of state[2]. Voisin[13] (1959) has included multipole moment densities in the derivation of Maxwell's equations.

Allowance for retardation effects was not explicitly made by Mazur and Nijboer in their procedure of ensemble averaging. As early as 1919, Dällenbach[14] (1919) had stressed the desirability of obtaining a covariant expansion of the polarization in terms of the atomic multipole moments. Defining the multipole moments in the momentary atomic frames, and then applying a Lorentz transformation to the reference frame, de Groot and collaborators[15] (1965) extended the derivation of Maxwell's equations into the relativistic realm. To this end, they devised a covariant technique of ensemble averaging in what they termed as a 'fluxion space' - i.e. a space spanned by the atomic positions, multipole moments and their various order time derivatives. The quantal treatment of Schram was extended by Crowther and ter Haar[16] (1971) to include the