

SYNOPSIS

Mixed boundary-value problems have been the subject of much investigation in recent years. The methods employed are: (i) method of subdivision into regions with continuous boundary conditions; (ii) use of function theory; (iii) use of integral equations. The last of these methods is used in the present work to solve the plate problem for rectilinear boundaries. Solutions are obtained for certain triangular plates.

The work is divided into three parts.

The first part deals with the problem of bending of thin plate, partly clamped and partly supported on its contour. We are here to solve the Lagrange's plate equation - the nonhomogeneous plate equation - under the boundary conditions that the deflection and the slope vanish on the clamped part and that the deflection and moment vanish on the supported part. The deflection is obtained by superposing two components - one due to the pressure normal to its plane and the other due to the clamping moment. The secondary solution i.e. that due to the clamping moment is represented in an integral form with the integrand as the product of the unknown clamping moment and the Green's function. The Green's function is here the unit impulse solution, the solution of the homogeneous plate equation with supported

boundary conditions, the plate being loaded by a unit normal edge moment. This is obtained by using the following analogy: the Laplacian of the deflection of a supported rectilinear plate under an isolated load corresponds to the stream function of a liquid due to a vortex. The boundary condition along the clamped part leads to a nonhomogeneous Fredholm integral equation for the moment. This is solved numerically or by reducing it to an infinite set of simultaneous equations. The cases of three triangular plates are treated in detail. Numerical results are obtained in all cases. Taking the plate areas to be equal it is seen that: (i) the maximum moment increases with the increase in the angle at the extremity of the clamped edge, but in all cases, is less ^{than} that in the case of a square; (ii) the point where it occurs is nearer to the greater of the angles.

In the second part two eigenvalue problems for the above boundaries under the same type of mixed conditions are treated. The problems of buckling under uniform lateral thrust and of the free vibrations of the plates are solved. The critical thrust and the frequencies of vibration are the eigenvalues.

The deflection is again represented in an integral form. The component due to the primary solution now drops out. The integral equation is homogeneous. The solubility condition leads to the characteristic equation. Reducing the integral equation, as before, to an infinite system of simultaneous equations, now homogeneous, the characteristic

equation is the vanishing of the infinite determinant of the system. This is the condition necessary for the existence of nontrivial solutions of the system. Numerical results show that: (i) the eigenvalues increase with the decrease in the angles at the ends of the clamped side; (ii) the relative rise in the parameter giving the first buckling load is greater than that which gives the fundamental frequency.

The last part deals with a thermo-elastic problem. There is a nucleus of thermo-elastic strain at a point of a plate. The solution is obtained by superposing on the thermo-elastic displacement-potential an Airy's stress function so as to satisfy the boundary conditions. The boundary conditions are taken as: part of the contour is stress-free and part is free from normal stress and tangential displacement. The Airy's stress function is represented in an integral form. The problem is reduced to the case treated in the first chapter and dealt with on similar lines.