Abstract

The thesis addresses the convergence of third order iterative methods used for finding roots of nonlinear equations in $\mathbb{R}$ and Banach spaces. Solutions of nonlinear equations are necessary due to their wide range of appearances in a number of fields. The boundary value problems appearing in Kinetic theory of gases, elasticity and other applied areas are reduced to solving nonlinear equations. Dynamic systems are mathematically modeled by difference or differential equations, and their solutions usually represent the equilibrium states of the systems obtained by solving nonlinear equations. Many optimization problems also lead to these equations. For example, the locations of the extremal points of a function require finding the zeros of the derivatives of that function. There are other areas such as chemical engineering, transportation, operation research and many others which involve solving nonlinear equations either individually or collectively. Iterative methods are used to solve these nonlinear equations. In general, it is not easy to select the starting point for an iterative method and hence the convergence of the method may not always be ensured. In order to establish the convergence of these methods many researchers gave sufficient conditions on the operator involved. Kantorovich gave sufficient conditions in order to study convergence of Newton's method. These sufficient conditions are known as Kantorovich theorem. Similar types of sufficient conditions are also provided to study convergence of a wide range of other iterative methods. Many researchers also gave sufficient conditions to study convergence of some third order iterative methods such as Halley's method, Chebyshev's method, super-Halley method etc. The study of convergence of these methods are necessary as these methods are successfully used to solve some conservative systems such as stiff system of equations where quick convergence is necessary. The major contributions of the thesis can be summarized as follows.

The chapter 3 gives a third order methods used to solve real nonlinear equations. It contains the function and it's first derivative. This chapter also gives two derivative free third order iterative methods. By combining these methods with that of bisection method and regula-falsi method, we provided two improved methods which generate a sequence

of iterates and a sequence of enclosing intervals such that both the sequence of errors and sequence of diameters converge cubically to 0 simultaneously.

The chapter 4 is concerned with the problem of studying multiple roots of real nonlinear equations. First when the multiplicity of the root is known, we have modified a quadratically convergent Newton-like method in order to preserve it's order of convergence. We have also proposed an improved derivative free transformation that reduces multiple roots of the equation to simple roots. Here we have used a quadratically convergent derivative free method to compute simple roots of the new function and multiplicity of the root. We have also used an algorithm developed in previous chapter in order to enclose simple roots of the new function or in other words multiple roots of the old function.

The chapters 5, 6, 7 and 8 are concerned with the problems of studying semilocal convergence of a third order method and it's different version used to solve nonlinear operator equations in Banach spaces. The convergence analysis is given under different continuity conditions such as Lipschitz/Hölder/ω-continuity conditions on first/second Fréchet derivative of the operator. In each chapter the convergence is established based on a family of recurrence relations. A priori error bounds are also obtained for the methods considered.