Chapter 1

Introduction

A porous medium may be defined as a solid matrix containing holes either connected or non-connected, dispersed within the medium in a regular or random manner provided such holes occur frequently in the medium. The interconnected pore space is termed as the effective pore space and the whole of the pore space is called the total pore space. Some examples of natural porous media are soil, beach sand, wood, human lungs etc.

The porosity of a porous medium is defined as the fraction of the total volume of the medium that is occupied by pores. But the effective porosity is defined as fraction of the bulk volume of the material occupied by the interconnected pores. The permeability K of the porous medium is defined as the ease with which the fluid is made to pass through the medium under an applied pressure gradient. This is independent of fluid properties but depends on the geometry of the solid grains. The ratio $\frac{K}{\mu}$ gives the mobility of the fluid in the medium, where μ is the viscosity of the fluid.

As the structure of the porous medium is complicated, the volume averaging technique is used to discuss the flow in the medium. The average of the fluid velocity over a representative element of the medium is called the seepage velocity **v**. But the average of the fluid velocity over the pore volume of the element is called intrinsic average velocity **V**. These two are related as $\mathbf{v} = \phi \mathbf{V}$, where ϕ is the porosity of the medium. Depending on the physical situation considered, different flow models are used to depict the fluid motion in the medium. A brief introduction to some of the flow models is given below.

Equation of Continuity

For the fluid part, equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0 \tag{1.1}$$

where ρ is the density of the fluid. When the density of the fluid is constant, the above equation becomes $\nabla.\mathbf{v} = 0$. For the solid part, equation of continuity holds since the solid in the medium is stationary (when we consider consolidated solid structure).

Equations of motion

Darcy [5], in the year 1856, first gave the governing equation for fluid motion in a vertical sand column. In vector form it is written as

$$\mathbf{v} = -\frac{K}{\mu}(\nabla p - \rho \mathbf{g}) \tag{1.2}$$

where p is the pressure, $\mathbf{g}$ is gravity vector. The negative sign indicates that the fluid velocity is in the opposite direction of increasing pressure gradient. This model does not take inertia effects into consideration. At moderate and high velocities, inertia effects become appreciable, causing an increase in form drag due to which the fluid velocity gets reduced. Forchheimer [6] in the year 1901 proposed a correction to the Darcy law and it is given by

$$\nabla p = -\frac{\mu}{K}\mathbf{v} - \frac{c}{\sqrt{K}}\,\rho\ \mid \mathbf{v} \mid \mathbf{v} \tag{1.3}$$

Here c is a Forchheimer constant. However for flows in high porosity medium and with large shear rates Darcy law appears to be inadequate since it does not contain term involving velocity gradient. Brinkman [7] proposed a modification of Darcy law in the form

$$\mu^* \nabla^2 \mathbf{v} - \frac{\mu}{K}\mathbf{v} = \nabla p \tag{1.4}$$

where μ^* denotes the effective viscosity of the fluid and is given by $\mu^* = \mu(1 + 2.5(1 - \phi))$. Another model which considers both Forchheimer and Brinkman terms is

$$\nabla p = -\frac{\mu}{K}\mathbf{v} - \frac{c\,\rho}{\sqrt{K}} \mid \mathbf{v} \mid \mathbf{v} + \frac{\mu^*}{\phi}\nabla^2 \mathbf{v} \tag{1.5}$$

The validity and the limitations of these models are well discussed in Nield and Bejan [6].

In physical situations where porous liquid interface is considered, the fluid in the liquid region obeys Navier - Stokes equation and it is given by

$$\rho \frac{D\mathbf{V}}{Dt} \ = \ \rho \mathbf{f} - \nabla p + \mu \nabla^2 \mathbf{V} \tag{1.6}$$

where $\mathbf{V}$ is the fluid velocity and $\mathbf{f}$ is the body force and $\frac{D}{Dt}$ is the material derivative. Non - dimensionalizing the equation (1.7) using $x^* = \frac{x}{L}$, $y^* = \frac{y}{L}$, $u^* = \frac{u}{u_\infty}$, $v^* = \frac{v}{u_\infty}$, $t^* = \frac{t}{T}$, $p^* = \frac{Lp}{\mu u_\infty}$, where L is the characteristic length, u_∞ is the free stream velocity, for the steady flow the above equation becomes (after dropping the *)

$$Re\ \mathbf{V}.\nabla\mathbf{V} = -\nabla p + \nabla^2\mathbf{V} + \frac{Re}{Fr^2}\frac{\mathbf{f}}{g} \qquad (1.7)$$

Here $Re = \frac{u_\infty L}{\nu}$ is the Reynolds number, $Fr = \frac{u_\infty}{\sqrt{gL}}$ is the Froud number. When $Re << 1$, equation (1.6) reduces to the following form

$$\nabla p = \mu\nabla^2\mathbf{V} \qquad (1.8)$$

which is known as Stokes equation. Flows obeying Stokes equation are called creeping flows or viscous flows.

Heat transfer is defined as energy transferred by virtue of a temperature difference or gradient. There are three distinct modes in which heat transmission can take place: conduction, convection and radiation. Conduction refers to heat transfer between two bodies or two parts of the same body through molecular vibration. Convective heat transfer / mass transfer is the transport of energy / solute from a surface by both conduction and gross fluid movement. The conversion of the internal energy of a substance into radiation energy is referred to as radiation heat transfer. It propagates by means of electro - magnetic waves depending on the temperature and on the optical properties of the emitter. In a physical situation if the fluid motion is induced by some external agent (pump, blower, etc.) the process is called forced convection. If the fluid motion arises from external force fields, such as gravity, acting on density gradients induced by the transport process itself, it is called natural convection. In the forced convection, flow field can be solved independently and this can be used in the energy, concentration equations to obtain the non - dimensional temperature and concentration. Where as in the natural convection the governing equations become coupled. In the mixed convection the order of the magnitude of the buoyancy force is comparable with the externally maintained pressure drop to drive the flow. Flow, heat and mass transfer in porous media is an interesting area where there are extensive applications to different fields of science and engineering. In filtration studies, the main concern is to determine how fluid moves through the porous structure leaving behind unwanted material. The porosity of the system is continuously changing and altering the pressure drop characteristics of the system. In the study of seepage of water through river beds and investigation of underground water resources, various types of convection through porous media are encountered. A better understanding of the convective transport through porous media is necessary in several applications.

Energy equation

First law of thermodynamics is the energy equation. Two options exist for deriving the energy equation in porous media. One is the two - phase model and the other, which is widely used, is local equilibrium model. Using local