SYNOPSIS

The thesis deals with problems of flow in a class of incompressible non-Newtonian isotropic fluids (Reiner-Rivlin fluid) for which the most general relation between stress tensor t^i_j , and strain-rate tensor d^i_j can be reduced (with the help of Cayley-Hamilton theorem) to the form

$$t^i_j = F_0 \delta^i_j + 2\mu_v d^i_j + 4\mu_c d^i_\alpha d^\alpha_j ,$$

where F_0, μ_v, μ_c are material constants and, in general, may be functions of invariants of strain-rate tensor. In this work μ_v and μ_c are taken to be constants and are called coefficients of viscosity and cross-viscosity respectively.

The thesis is divided into two parts. The first part deals with the superposability of flows, which may be defined in the following manner(1):

Let $(\bar{V}_1, p_1, \Omega_1)$ and $(\bar{V}_2, p_2, \Omega_2)$ be two solutions of the equations

$$\frac{\partial \bar{V}}{\partial t} + (\bar{V}.\nabla)\bar{V} = \text{grad}\,\Omega + \frac{1}{\rho}\nabla.\bar{t},$$

and

$$\nabla.\bar{V} = 0,$$

with prescribed initial and boundary conditions, where $\bar{V}$ is the velocity vector, Ω is the extraneous-potential and $\tilde{t}$ is the stress tensor. The two solutions are said to be superposable if there exists a pressure π, such that

$$(\bar{V}_1+\bar{V}_2\,,\; p_1+p_2+\pi\,,\; \Omega_1+\Omega_2)$$

satisfies the above equations with necessary modifications in initial and boundary conditions. This definition has been used to find the conditions of superposability and self-superposability of non-Newtonian fluids. These conditions are different from those of Newtonian fluids[1] $(\mu_c = 0)$ for three-dimensional motions, but for two-dimensional motions the conditions are identical, except that the pressure π is modified by terms containing μ_c. It has been found that (i) all irrotational motions are self-superposable and any two motions of this type are superposable on each other; (ii) all Beltrami motions are superposable and self-superposable; (iii) any two motions are superposable on each other if the vortex lines of the first coincide with the stream lines of the other and the vortex lines of the second coincide with the stream lines of the first; (iv) the diffusion of vorticity in Beltrami flow does not depend on cross-viscosity and is governed by the diffusion equation

$$\frac{\partial \bar{\omega}}{\partial t} = -\nu_v \operatorname{curl}\operatorname{curl}\bar{\omega},$$

where $\bar{\omega}$ is the vorticity vector and ν_v is the kinematic coefficient of viscosity. Some examples of superposable flows are also given.

The second part treats the effect of μ_c, the coefficient of cross-viscosity, on the flow parameters, viz. the distribution of velocity, the boundary layer thickness and the wall stresses. An attempt has been made to explain some experimental results of Popper and Reiner(2) and Ward and Lord(3) on the basis of cross-viscosity. Four special problems have been considered.

(i) The equations of motion for the flow due to steady rotation of an infinite plate have been reduced to ordinary differential equations which can be integrated exactly after the manner of Cochran(4). For the sake of simplicity these equations have been integrated by Kármán-Pohlhausen method. It is found that the cross-viscous effect depends on a non-dimensional parameter $K = \mu_c \Omega / \mu_v$, where Ω is the angular velocity of rotation of the plate. In a viscous Newtonian liquid there is an axial flow toward the rotating lamina and the liquid moves radially outwards in the boundary layer region; but in the case of non-Newtonian fluids for large values of K there is a possibility of the flow pattern being completely reversed, i.e., the flow will be radially inwards and axially away from the plate. The thickness of the boundary layer increases with K. The frictional torque on a disc of radius c decreases with K.

From energy considerations an inequality has been deduced which restricts the possible types of flows in this fluid.

(ii) The equations of motion for the flow between two infinite parallel plates, one of which is rotating and the other is at rest, have been approximately solved. It is found that under certain conditions, depending on the distance d^* between two plates and the velocity of rotation Ω, the plate at rest experiences a suction; but if d^* is decreased and Ω is increased sufficiently the non-rotating plate experiences pressure which increases with Ω and μ_c. For a particular liquid and speed of rotation the normal thrust on the non-rotating plate varies as d^{*-2} but becomes appreciably constant when d^* exceeds a certain value. These results hold good for small values of d^*. Ward and Lord(3) took the solution of commercial polyisobutylene in mineral oil and sheared it between a stationary inner cylinder and a rotating outer cylinder. They found that the vertical thrust on the base of the inner cylinder decreased with the increase of spacing between the bases of the cylinders. This fact is explained by the above theoretical investigation. Popper and Reiner(2) sheared air between two discs, one rotating and the other at rest. They found that, upto a certain value of the distance d^* between the plates and the speed of rotation the manometer at the centre of the non-rotating disc indicated a suction; but when d^* was decreased and Ω was increased sufficiently it recorded a pressure of about half an atmosphere (centripetal pump effect). Reiner[5] conjectured that the centripetal pump effect in his experiment was due to air being a Maxwellian liquid (elasto-viscous). Taylor[6] explained these effects on the basis of compressibility of air and also due to non-parallelism of plates. The theoretical work reported here also explains this effect on the basis of cross-viscosity.

(iii) The equations of motion, when an infinite plate performs rotatory oscillations in non-Newtonian fluids, have been integrated approximately. It is observed that in Newtonian fluids the axial flow at infinity is of oscillatory type and the effect of cross-viscosity is to increase the amplitude of the oscillation of the axial velocity at infinity.

(iv) The equations of motion for the steady flow of a non-Newtonian liquid near a stagnation point have been reduced to ordinary differential equations which have been integrated by Kármán-Pohlhausen method. The effect of cross-viscosity depends on a non-dimensional paramter $K = \mu_c a / \mu_v$, where a is constant depending on the velocity of the potential motion and has the dimensions $[T^{-1}]$. The boundary layer thickness decreases with K upto $K \simeq 0.15$ and then increases rapidly. The shearing stress at the wall decreases with the increase of K while the normal stress at the wall is independent of viscosity as well as cross-viscosity. Incidentally it may be noted that the shearing stress at the wall also decreases in a similar two-dimensional problem in magnetohydrodynamics[7]. It is probable that some effects which arise due to hydromagnetic interaction may also be present in non-Newtonian fluids. The possible types of flows have been restricted by an inequality derived from energy considerations. It appears that the cross-stresses which come into being in a non-Newtonian fluid modify the flow considerably.

References:

(1) Ballabh, R.; 1940, Proc.Benares Math.Soc., Vol.5, p.70.

(2) Popper, R. and Reiner, M.; 1956, British Jour.App.Phys. Vol.7, pp. 452-453.

(3) Ward, A.F.H. and Lord, P.; 1953, Proc.Sec.International Cong. on Rheology (Butterworth Scientific Publication, London), pp.214-220.

(4) Cochran, w.; 1934, Proc.Camb.Phil.Soc., pp.360.

(5) Reiner, M.; 1957, Further Research on Physics of Air Viscosity (Technical Report).

(6) Taylor, G. and Saffman, P.G.; 1957, Jour.Aero.Sc., Vol.24, pp.553-562.

(7) Neuringer, J.L. and William, M.; 1958, Jour.Aero.Sc., Vol.25, No.3, pp.194.