# Chapter 1

# Introduction

The thesis is devoted to study of some problems of momentum and heat transfer in incompressible viscous flows of electrically nonconducting and conducting fluids in the presence of magnetic field. The effects of suction or blowing on momentum and heat transfer characteristics in some of these flows are studied in detail. The influence of suction or blowing on momentum and heat transfer in flows of certain non-Newtonian fluids is also investigated. Apart from the study of these flow problems, an analysis is also made of the effects of throughflow on the thermal instability of a horizontal layer of an incompressible viscous electrically conducting fluid uniformly heated from below in the presence of a vertical magnetic field. Before we discuss the motivation and the scope of these problems, we present an overview of literature which has bearing on the contents of the thesis.

## 1.1 Incompressible viscous flow past bodies subject to suction or blowing

It is well known that the flow in a boundary layer separates in regions of adverse pressure gradient. The formation of separation has several undesirable effects in so far as it leads to increase in the drag on the body immersed in the flow and adversely affects heat transfer from the surface of the body. Separation can be prevented by suction since the low energy fluid in the boundary layer is removed. Further, suction also tends to stabilize the boundary layer flow.

Suction can be applied to the bodies in two ways. If the surface of the body is permeable and can therefore let the fluid through, the boundary layer can be controlled by suction. On the other hand suction can also be applied through narrow slits on the surface. Prandtl [1] showed that in the flow past a circular cylinder where suction is applied to one side through a narrow slit, the flow follows along the surface of the body where the suction is applied for considerable distance thus preventing separation. In consequence of this, the drag is considerably reduced. The steady flow of a uniform stream of an incompressible viscous fluid over an infinite porous plate subject to uniform continuous suction was studied by Griffith and Meredith [2]. The steady flow in the boundary layer on a semi-infinite flat plate at zero incidence with a uniform stream was investigated by Iglisch [3], the plate being subject to uniform continuous suction. Experimental investigations by Kay [4] confirmed the theoretical results of Iglisch.

Another way of preventing separation consists of supplying additional energy to the fluid particles which are low in energy in the boundary layer. This can be accomplished by tangentially blowing higher velocity fluid out from inside the body. The wall shear stress and hence the friction drag is reduced by blowing. The stability of the boundary layer and transition to turbulence are also significantly influenced by continuous suction and blowing. Zien [5] presented an approximate method for the calculation of heat transfer in boundary layer flows past bodies subject to suction or blowing. Gersten and Körner [6] studied momentum and heat transfer in the steady flows of an incompressible viscous fluid in the laminar boundary layer over a porous wedge subject to suction or blowing. Comprehensive summaries of research in the area of boundary layer control by suction or injection in the flow of a viscous fluid are given in Lachmann [7] and Chang [8].

## 1.2 Flow in the Ekman layer

Let a large body of an incompressible viscous fluid at rest relative to uniformly rotating axes be set into motion by a uniform gradient of modified pressure which is then balanced by the Coriolis force. A steady state with uniform velocity then prevails and this flow is known as geostrophic flow. If in addition, the fluid is bounded by a horizontal rigid plane at rest relative to the rotating axes, then the departure from the uniform stream takes place in a layer near the rigid plane. This layer in which viscous forces are balanced by the Coriolis forces is known as the Ekman layer. The steady flow in this layer was first noticed by Ekman [9], and was used in a discussion of wind-generated ocean currents on rotating earth. An Ekman layer flow below a geostrophic flow of variable velocity has an additional property, which leads to a significant interaction with the geostrophic region. There generally arises a transverse motion in the associated Ekman layer on the rigid boundary. This transverse motion involves divergence (or convergence) and thus gives rise to a flow out of (or into) the geostrophic region. This process in known as Ekman layer suction (or injection). The suction changes the boundary condition experienced by the geostrophic region. Because of the Taylor-Proudman theorem (Chandrasekhar [10]), this has an effect throughout the geostrophic flow. Transfer of fluid between boundary layers and the body of the fluid in this way is the process (known as spin-up) by which a tank of fluid is brought to the angular velocity of its boundaries (Greenspan and Howard [11]). Thornley [12] investigated the flow generated in a semi-infinite expanse of an incompressible viscous fluid bounded by an infinite non-porous flat plate when both the fluid and the plate are in rigid-body rotation and additionally the plate performs non-torsional oscillations in its own plane. Gupta [13] studied the steady flow in an Ekman layer on an infinite porous plate subjected to suction or blowing.

## 1.3 Flow over a rotating disc

A plane disc of very large diameter rotates in its own plane with a uniform angular velocity about an axis normal to the plane in an incompressible viscous fluid which is initially at rest everywhere. The relative motion of disc and the fluid sets up viscous stresses which tend to drag the fluid round with the disc. An exactly circular motion of fluid near the disc is not possible, since there is no imposed radial pressure gradient to provide the inward radial acceleration and the fluid near the disc spirals outward. This outward radial motion must be compensated by an axial motion towards the disc in order to satisfy the conservation of mass and in this way the vorticity generated near the boundary is prevented from spreading far from it.

The steady motion of an incompressible viscous fluid due to an infinite rotating disc was first investigated by von Kármán [14] who gave similarity solution for the velocity distribution. Numerical solutions of the governing Navier-Stokes equations for the above flow was given by Cochran [15]. Rogers and Lance [16] investigated the steady flow of an incompressible viscous fluid over an infinite disc rotating with a uniform angular velocity about an axis normal to the disc when the fluid at infinity also rotates with a different angular velocity about the same axis. Stuart [17] studied the effect of uniform suction at the disc on the steady flow due to a rotating disc. Gregory, Stuart and Walker [18] investigated the stability of flow over a rotating disc with or without suction. The influence of blowing through a porous rotating disc on the flow induced by the disc was analyzed by Sparrow and Gregg [19] and Kuiken [20]. The problem of boundary layers in rotating flows is very much complicated by the added effect of translating boundaries which destroys the axisymmetric character of the basic flow. Rott and Lewellen [21] gave exact solutions of the Navier-Stokes equations which represent certain flows in which an infinite disc and the adjacent fluid have relative rotation and translation. He gave numerical solutions for two cases of steady flow in which either the fluid is in uniform translation and the plate is rotating, or the plate is in pure translation adjacent to a rotating fluid. Wang [22] studied the effect of uniform shear on the flow of an incompressible viscous fluid caused by the uniform rotation of an infinite disc about an axis normal to the disc.

## 1.4 Oblique stagnation-point flow

A stream of an incompressible viscous fluid impinges on a wall at right angles to it and flows away from a point on the wall. We refer to this flow in the neighbourhood of the wall as 'stagnation-point flow'. Oblique stagnation-point flow occurs when a jet of viscous fluid impinges on a surface obliquely. It is also observed in the region where a separated viscous flow reattaches to a boundary. The knowledge of the flow structure around the point of stagnation is of biomedical importance, especially for studying the pathogenesis of atherosclerosis and thrombosis. Stuart [23] first gave the solution for steady oblique stagnation-point flow of a viscous fluid impinging on a flat plate, a result later rediscovered by Tamada [24] and Dorrepal [25]. Exact similarity solutions for the impingement of two viscous immiscible oblique stagnation flows forming a flat interface were given by Tilley and Weidman [26].

## 1.5 Flow of non-Newtonian fluids past external surfaces

The increasing emergence of non-Newtonian fluids, such as molten plastics, emulsions, pulps, polymer melts, high viscosity silicone oils as important raw materials and products in a wide variety of industrial processes, has stimulated a considerable amount of interest in the behaviour of such fluids when in motion. In particular, what has been studied very intensely for obvious practical reasons is how momentum and heat are transferred to a moving non-Newtonian fluid under the more common flow configurations usually met in practice. It is, therefore, understandable that some of the simpler problems of classical hydrodynamics such as pressure drop and heat transfer in channels and pipes, flow between two rotating concentric cylinders, Couette flow between two parallel plates etc. have been reinvestigated in non-Newtonian fluids. The term non-Newtonian fluid is one of very great generality and includes all fluids for which the equations of motion of Newtonian fluid (i.e., viscous fluid characterized by the property that during its motion the stress is proportional to the rate of strain) do not apply. There exist two different classes of non-Newtonian fluids.

(i) Power-law fluids : These fluids are characterized by the property that during its motion the stress is a nonlinear function of the rate of strain. Such fluids with anomalous viscosity are inelastic in so far as they neither show stress relaxation nor normal stress effects.

(ii) Second-grade ( or second-order) fluids: These fluids are characterized by the property that they exhibit normal stress differences in a shear flow and hence these fluids are viscoelastic.

The results of these studies are given in the review article by Metzner [27] and by Lyche and Bird [28], Dodge and Metzner [29], and Mishiyoshi [30].

The boundary layer equations governing the flow of non-Newtonian viscoplastic fluids were derived by Oldroyd [31]. Two and three-dimensional boundary layer equations governing momentum and heat transfer in flow of a non-Newtonian viscoinelastic fluid with anomalous viscosity were developed by Schowalter [32], Acrivos et al. [33] and Shul'man [34]. Acrivos et al. [35] presented a theoretical analysis of forced convection momentum and heat transfer in laminar boundary layer flows of non-Newtonian fluids past external surfaces. The corresponding analysis of laminar natural convection heat transfer to non-Newtonian fluids was given by Acrivos [36].

The steady flow of a uniform stream of an incompressible viscous fluid over an infinite porous flat plate subject to uniform suction was investigated by Griffith and Meredith [2]. The velocity distribution for steady flow over a large but finite plate with constant suction will approximate to the velocity distribution obtained by Griffith and Meredith at points far away from the leading edge. Comprehensive summaries of research in the area of boundary layer control by suction or injection in the flow of a viscous fluid can be found in the books by Lachmann [7] and Chang [8]. Gersten and Körner [6] investigated the steady flow of an incompressible viscous fluid in the laminar boundary layer over a porous wedge subject to suction or blowing. Taking frictional heat into account, they determined the temperature distribution in the flow and wall heat flux in the case of a variable wall temperature distribution. Heat transfer by natural convection flow of viscous fluid past hot surfaces subject to suction or blowing was studied by Eichorn [37], Sparrow and Cess [38] and Merkin [39]. Shear flow of a viscoelastic fluid past a flat plate subject to suction was analyzed by Gupta [40]. Kaloni [41] investigated the fluctuating flow of a viscoelastic fluid past an infinite porous plate subject to uniform suction. The corresponding problem of flow of a viscoelastic fluid past an infinite plate with variable suction was analyzed by Soundalgekar and Puri [42]. The steady flow of an incompressible non-Newtonian fluid past an infinite porous plate subject to either suction or blowing was investigated by Rajagopal and Gupta [43].

## 1.6 Shear flow past a flat plate

In the study of high-speed viscous flow past a two-dimensional blunt body, it is usually necessary to consider a curved shock wave formed near the body. Consequently there exists a region of large entropy gradients and high vorticity between the shock wave and the boundary layer. Such a situation arises, for example, in the study of hypersonic viscous flow past a flat plate. It may be noted that this situation is somewhat different from Prandl's boundary layer problem. According to the classical boundary layer theory, the flow about bodies at high Reynolds number can be considered as consisting of two regions: an outside inviscid irrotational flow and a thin boundary-layer region adjacent to the body. This point of view leads to the approximation, that on a slightly curved surface the pressure gradient normal to the surface is negligibly small. The additional assumption that the inviscid flow is irrotational leads to the requirement that the shear stress is zero at the outer edge of the boundary layer. In this theory any interaction between the two regimes can be accounted for by a simple correction to the body shape based on the boundary layer displacement thickness. However, in connection with hypersonic boundary layers, this classical point of view has been modified to take into account the fact that the inviscid free stream is rotational (Ferri and Libby [44]). In such situations an interection between the two regimes leads to a self-induced axial pressure gradient. Using boundary layer approximations, simple shear flow past a semi-infinite flat plate in an incompressible fluid of small viscosity was studied by Li [45] and Yen [46]. Steady shear flow of an incompressible viscous fluid past an infinite porous flat plate subject to uniform suction was investigated by Sakurai [47].

## 1.7 Thermal instability of a layer of viscous fluid heated from below

If a layer of viscous fluid contained between two horizontal boundaries is heated from below in a field of gravity, the system becomes top-heavy. This is due to the fact that on account of thermal expansion, the fluid at the bottom becomes lighter than the fluid at the top. This top-heavy arrangement is potentially unstable. Because of this instability, there will be a tendency on the part of the fluid to redistribute itself and remedy the weakness in the arrangement. However this natural tendency of the fluid is inhibited by viscosity of the fluid. This implies that the adverse temperature gradient which is maintained due to heating from below must exceed a