ABSTRACT

Radial basis functions (RBFs) have become a vital tool for multivariate interpolation and meshless methods due to their ability to handle arbitrarily scattered data, straightforward generalization in higher dimensions, and spectral convergence. Using RBFs in interpolation and meshless methods, however, often leads to solving an ill-conditioned system of linear equations, which has been a crucial concern in past two decades. As an alternative approach for circumventing the above-mentioned ill-conditioning problem, in this dissertation, a novel radial basis function has been proposed through the hybridization between an infinitely smooth Gaussian kernel and a piecewise smooth cubic kernel. In order to select the 'optimal' values of the shape parameter of the Gaussian kernel and the weights controlling the Gaussian and the cubic part in the hybrid kernel, an algorithm using global particle swarm optimization has been proposed. Through a series of numerical tests with synthetic as well as real data, it has been demonstrated that such hybridization stabilizes the interpolation scheme by yielding a superior implementation compared to those obtained by using only the Gaussian or cubic kernels. Further, the hybrid RBF was used to develop an improved version of a global meshless method *viz.* radial basis-pseudospectral method (RBF-PS). It was observed that the proposed approach significantly reduces the ill-conditioning problem in the RBF-PS method, and at the same time, it preserves the stability and accuracy at small shape parameters. Further, the hybrid RBF was used to develop a local meshless method *viz.* radial basis-finite difference method (RBF-FD). The improved RBF-FD approach was further used to develop a general purpose 2D Helmholtz solver, which did not have 'pollution-effect' and worked effectively at all wavenumbers. The developed Helmholtz solver was used for numerical solution of frequency-domain acoustic wave equation coupled with absorbing boundary conditions and compared with the standard FD methods. Since the hybrid kernel provides a well-conditioned formulation in the RBF-direct framework, its computational time is significantly less than the available 'stable' approaches. The algorithms and results presented in this thesis can further be utilized to develop efficient numerical modeling tools in computational geoscience as well as in other fields.

Keywords: *Radial basis functions; ill-conditioning; scattered data interpolation; Meshless Methods; Computational wave propagation*;