Chapter 1

Introduction

1.1 General

Shells made of laminated composites are known to be desirable structural elements in various applications associated to the advanced areas in aerospace, automotive, civil and marine industries. This is due of the fact that the laminated composites meet many of the desirable requirements like, higher strength-to-weight ratios, better corrosion resistance, longer fatigue life and greater stealth characteristics over metals along with the tailorable directional properties of the stiffness coefficients. For the last few decades a great deal of research effort has been invested mainly on the development of shell models that adequately describe their complex structural behavior under different situations. The inherent heterogeneity and anisotropy of laminated composite shells as evidenced in the stacking of several dissimilar layers and in the strong discontinuity in material properties across the interface, make both the *classical laminate theory*(CLT) and the *first order shear deformation theory*(FSDT) inadequate in furnishing satisfactory information about the global structural behavior (i.e., transverse deflections, natural frequencies and the stress resultants). More importantly, the laminated composite shells show relatively weak rigidity in the transverse shear (i.e., $E_T/G_{LT} \approx E_T/G_{TT} \approx 10 - 200$) making the interfaces in a laminated composite shell susceptible to failure even for thin structures. Thus, the estimation of the shearing stresses at the lamina interfaces (i.e., the local behavior) becomes necessary for the design of important structures made of composite shells.

The laminated composite shells with the embedded or surface bonded piezo-

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electric materials, in the form of layers or patches, provide the so-called new generation **smart structures**. The superior performance advantages of these multilayered smart shell structures over the conventional composite shells are mainly due to their ability to store or convert the internal energy in different forms (e.g., mechanical to electrical energy) that combine the superior mechanical properties of composite materials along with the capability to sense and modify their mechanical behavior. Piezoelectric materials are widely considered as smart or active elements due to their frequency bandwidth, low cost, fast energy conversion ability. It can also be easily shaped to any size for surface bonding or embedding into a composite substrate. There are two classes of piezoelectric materials mostly used are ceramics and polymers. The best known piezo-ceramic is the Lead Zirconate Titanate (PZT), which has a recoverable strain of 0.1% and is widely used as actuator and sensor for a wide range of frequencies, including ultrasonic applications. The best known piezo-polymer is the Poly Vinylidene Fluoride (PVDF), which is mainly used as sensors.

The electro-elastic laminated shells show significant interactions between the elastic and electric fields due to the coupled nature of the constitutive equations. The various couplings in the constitutive relations and the presence of curvature of the shell geometry make the behavior of these smart shell structure quite complex. Also, the order of the magnitude of the elastic constants are several hundred times higher than those of the electric constants. This, in fact, offer a great challenge in modeling the elasto-electric interactions for practical applications. Though the mechanics of multi-functional materials are understood reasonably well, rigorous modeling using these materials in a composite structural system under different loading conditions is still a challenging task. Due to the limited applicability of rigorous analytical techniques in many situations, the approximate solutions using finite elements have been preferred for solving the engineering problems. Therefore, there is a strong need for the development of the theory and the modeling of the multi-field (piezo-elastic) coupled interaction in smart shells.

1.2 Review of Literature

Modeling of laminated composite shells is a wide area of research. Over the years, numerous studies have been carried out and are reported in this area. The different attempts made by various investigators to model the multilayered

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composite shells and the coupled electro-elastic field problem of smart composite shells may be broadly classified under the following heads:

(a) Multilayered Composite Shells

- Equivalent Single Layer(ESL) Model
- Layer Wise(LW) Model and Three Dimensional Model
- Refined ELS combined with Zig-Zag(ZZ) kinematics

(b) Coupled Piezo-Elastic Smart Composite Shells

- Coupled ESL Model
- Coupled LW Model and Three dimensional Model
- Coupled Refined ESL combined with Zig-Zag(ZZ) kinematics

1.2.1 Multilayered Composite Shells

1.2.1.1 Equivalent Single Layer (ESL) Model

The *equivalent single layer*(ESL) model are those in which a heterogeneous laminated shell is treated as a statically equivalent, single layer having complex constitutive behavior, degenerating the 3-D continuum problem to a 2-D one. The ESL model for laminated composite shells can be based on *classical laminated shell theory* (CLST), *first order shear deformation theory* (FSDT) and refined or *higher order shear deformation theory* (HSDT).

Love (1888, 1927) established the foundation of classical shell theory. The proposed theory is based on *Kirchhoff* hypothesis (i.e. fibers normal to the mid-surface remain normal and inextensible during deformation) and often referred as *Kirchhoff-Love shell theory* where, the transverse normal and shear effects are neglected i.e., entire deformation is due to bending and in-plane stretching. A number of theories have been developed in linear domain for thin elastic shells incorporating Love-Kirchhoff hypothesis (e.g., Donnell (1938), Vlasov (1949), Sanders (1955), Koiter (1960), Naghdi (1963)). Subsequently, a number of theories have been developed for thin laminated composite shells incorporating Kirchhoff-Love hypothesis (see, e.g., Ambartsumyan (1953, 1966) and Dong *et al.* (1962)). Surveys of various classical shell theories can be found in the works of Naghdi (1971) and Bert and Francis (1974). Soldatos (1984) made a numerical comparison of

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several classical theories for the free vibration of laminated cylindrical shells. Numerous works are available in this topic and have not been mentioned here for the sake of brevity.

It has been observed that the laminated composite shells exhibit relatively weak rigidity in the transverse shear making the interfaces in a laminate susceptible to failure even for thin structures. Therefore, the estimation of the shearing stresses at the lamina interfaces (i.e., the local behavior) becomes important for the design of important structures. Reddy and Kuppusamy (1984) have shown that the natural frequencies predicted by the classical laminated plate theory (CLPT) may be as much as 25 percent higher than those predicted by including the shear effects for a plate with side to thickness ratio of 10. Moreover, the classical shell theory under-predicts deflections and over-predicts natural frequencies and buckling loads. In addition, it has been realized that the FE models based on the classical shell theory require C^1-continuity of the transverse displacement, which complicates the development of conforming elements and restrict their use.

The most simplified model that includes the effect of transverse shear deformation is known as the *first order shear deformation theory* (FSDT)(Reissner (1945), Mindlin (1951)). FSDT is the next theory in the hierarchy of ESL theory for laminated composite shells. A Mindlin-type first order transverse shear deformation theory (FSDT) has been first developed by Whitney and Pagano (1970) for multi-layered anisotropic plates, and Dong and Tso (1972) for multi-layered anisotropic shells. FSDT assumes the constant shearing strains through the thickness of the shell. This implies the piecewise constant nature of the transverse shearing stresses through the thickness of the shell and that is a direct contradiction with the 3D elasticity theory (Pagano (1969, 1970*a*)). This necessitated the introduction of an arbitrary *shear correction factor*(see, e.g., Whitney (1969*b*,*c*, 1973)) which depends on the lamination parameters for arbitrarily laminated composite structures. A number of closed form and analytical solutions (see, e.g., Bert and Kumar (1982), Bert and Reddy (1982), Reddy (1984*a*), Khdeir *et al.* (1989), Librescu *et al.* (1989*a*,*b*)) and FE models (see, e.g., Rath and Das (1975), Noor and Mathers (1975), Reddy (1982), Kumar and Rao (1988), Noor and Peters (1989)) have been developed based on FSDT for laminated composite shells. FE models for laminated composite shell based on FSDT are economical in terms of the number of degrees of freedom used. Also, finite elements based on the FSDT have the advantage of requiring only C^0-continuity

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of all primary variables. However, they can exhibit spurious transverse shear stiffness (or locking) as the shell becomes thin. Further, it has been observed that with the FSDT approximation, the estimated transverse stresses are substantially far from their actual values. Therefore, the incorporation of the warping of the cross-sections, normal to the middle-surface of the shell, has become inevitable to overcome these shortcomings of the FSDT particularly for the moderately thick laminated shell.

While the laminated plate models based on the HSDT are vast in literature but shell models are only a few. In many cases simplified shallow shell theory has been developed based on assuming a displacement field of polynomial form having degree more than unity. Whitney and Sun (1973, 1974) developed a shallow shell theory for the static deformation of anisotropic cylindrical shells based on a displacement field in which the middle surface displacements are expanded as linear functions of the thickness coordinate and the transverse normal displacement is expanded as a quadratic function of the thickness coordinate. This theory is computationally more demanding as higher order power of thickness coordinate involved. However, the use of the shear correction factor were made as the theory assume constant shear stresses through the thickness. Bhimaraddi (1985) presented a higher-order displacement model for dynamic analysis of laminated cylindrical shell. The displacement model implicitly satisfied the traction free surface condition at the bounding surfaces of the shell, so the number of variables remains five. Simple HSDT with a cubic variation of the in-plane displacements and a constant transverse displacement through the thickness is due to Levinson (1980), Murthy (1981) and Reddy (1984*b*) for composite plates and by Reddy and Liu (1985) for laminated shells. The theory satisfies the traction free conditions at the bounding surfaces and does not require any shear correction factors. The main advantage of this theory is its simplicity in accounting for a cubic variation of the in-plane displacements through the thickness while keeping the number of unknown variables to be same as is considered in the FSDT. The theory has been extensively used for computation of global behavior (deflection, natural frequencies, buckling load etc) of laminated shells (see, e.g., Simitses and Anastasiadis (1992), Huang (1994)) and also for the prediction of transient response of cylindrical shell (Khdeir *et al.* (1990)).

Mallikarjuna and Kant (1990, 1992) developed a super-parametric higher order shell element for laminated composite and sandwich structure based on higher

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order displacement model and 3-D Hooke's law for shell material. Lim and Liew (1995) and Liew and Lim (1996) presented higher-order shear deformation theory for free vibration of shallow shells based on third-order expression for in-plane, transverse and rotation displacement fields. The derivation yields cubic and unsymmetric strain distribution through shell thickness. Bending behavior and free vibration laminated composite shells with/without a cutout has been studied by Sai Ram and Sreedhar Babu (2001*a*,*b*, 2002) using the finite element method based on a higher-order shear deformation theory by assuming constant transverse displacement through the thickness. Semi-analytical post-processing approach for prediction of interlaminar shear stress distribution through the thickness of laminated general shell are due to Chaudhuri and Seide (1987) and Chaudhuri (1988, 1990). However, they advocated a two-step procedure, a simplified analysis followed by another semi-analytical post-processing part. The post-process part in this group has been further improved by Cho and Kim (1996) who utilize the displacement field of a efficient higher order theory(EHOSD).

Touratier (1992*a*,*b*) presented a theory for laminated composite shallow shells which accounts for assumed harmonic distribution of the transverse shearing strains through the thickness of the shell and tangential stress-free boundary condition on the bounding surfaces of the shell. The theory contains the same independent displacement fields as in the shear deformation theory. This theory has been extended by Ossadzow and Touratier (2001) for doubly curved composite shells in which the continuity conditions for displacements and shear stresses at layer interfaces and on the boundary surfaces of the shell are exactly satisfied. The refined transverse shear and membrane deformation have also been represented by trigonometric function. A simple C^0 higher-order facet shell element-based on a shear deformable model of higher-order theory has been presented by Khare *et al.* (2004) for the free vibration analysis of isotropic, orthotropic and layered anisotropic composite and sandwich laminates. where, non-uniform distribution of displacements through the shell thickness has been incorporated, and shear correction coefficients were eliminated.

1.2.1.2 Layer Wise(LW) and Three Dimensional Models

Closed form elasticity solutions are possible in a very few special situations. Works under this have been carried out by Pagano (1969, 1970*b*) for laminated plates. The effect of boundary conditions in vibration and buckling responses of

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composite plates has been investigated by Whitney (1969a). Varadan and Bhaskar (1991) and Ren (1987) developed the exact solution of laminated cylindrical shell.

Approximate but a close compromise with the 3D elasticity theory is the layer-wise theory(LW). The layer-wise theories are based on the assumed variation of either displacements or/and the stresses in the thickness direction of the composite plates/shell within each lamina. Thus, the numbers of unknowns are dependent on the total number of layers of the plates/shell unlike a layer independent fixed number in case of equivalent single layer theories(ESL). In general, the order of the governing equations in these theories is dependent on the number of layers of the plates/shell.

The simplified LW models have been developed by Zukas and Vinson (1971), Epstein and Glockner (1971), Grigolyuk and Chulkov (1972), Librescu (1974), Alam and Asnani (1984a,b), Reddy *et al.* (1989) and Noor *et al.* (1989) which are purely based on kinematic hypotheses. These theories are based on piecewise linear variation of the in-plane displacements and constant transverse displacement through the thickness (i.e., the transverse shear strain is piecewise constant, while the transverse normal strain is zero through the thickness). Since the transverse shear stresses are constant within each layer, the continuity of these stresses could not been satisfied at layer interfaces also normal displacement does not have a layer-wise representation. But compared to the ESL models, these theories provide a more realistic description of the kinematics of composite laminates by introducing discrete layer transverse shear effects into the assumed displacement field. To overcome the drawback in displacement based layer-wise theory a number of theories have been developed based on a hybrid combination of kinematic and stress hypotheses. The continuity of the transverse stresses at layer interfaces have been satisfied (see, e.g., Hsu and Wang (1970, 1971), Rath and Das (1973)) by using explicit approximations for the transverse stresses within each layer. But, these models are computationally more intensive than their displacement based counterparts.

LW theory based on layer-wise expansion for all displacement components have been developed by Epstein and Huttelmaier (1983), Robbins and Reddy (1993) and Reddy (1987), for plates and by Pinsky and Kim (1986), Basar and Ding (1995) for shells. Barbero and Reddy (1990) presented a layer-wise generalized laminated cylindrical shell theory in which any desired degree of approximation of the in-plane displacements can be achieved by a suitable selection of the variables

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and functions. Therefore, with the flexibility in the choice of the interpolation functions to approximate the in-plane displacements can be accommodated although at the expense of increasing the number of variables.

Recently, Carrera (2000); Carrera and Demasi (2002*b*,*a*) developed several LW displacement models and layer-wise mixed models, assuming Legendre polynomial approximations in the thickness direction ranging from linear to fourth degree. In the layer-wise mixed model, the displacement field and transverse stress components are approximated through the thickness of each lamina. The LW mixed models with linear and quadratic approximation through the thickness provide better results than the corresponding LW displacement ones. However, for approximations of cubic and fourth degree, no significant difference is observed between the LW mixed and displacement models.

Although, layer-wise theory can adequately describe the deformation behavior through the thickness yet it is not computationally efficient since it employs a large number of degrees of freedom depending upon the number of layers.

1.2.1.3 Refined ESL Combined with Zig-Zag(ZZ) Kinematics

Multilayered shells are subjected to discontinuity of physical/mechanical properties in the thickness direction. Two-dimensional (2D) modeling of multi-layered plates and shells require a piecewise continuous displacement field in the thickness direction and need the fulfill interlaminar continuity of transverse shear stresses at each layer interfaces. The piecewise form of transverse stress and displacement fields are often referred in the open literature as *zig-zag* (ZZ) and *interlaminar static continuity*(IC), respectively. The theories which describe these two effects are referred to as *zig-zag* theories.

Lekhnitskii (1935) proposed a theory based on the zig-zag form of both the longitudinal and transverse displacements. The longitudinal displacements have been assumed to be cubic distribution whereas the transverse displacement have been chosen to be parabolic in the z direction. Lekhnitskii's work was originally presented as an elasticity solution for layered, cantilever beams. Ren (1986) extended Lekhnitskii 's theory to orthotropic and anisotropic plates. The comprehensive review on the earlier development of Zig-Zag theories including many references were reported by Carrera (2003).

Di Sciuva (1987) proposed a zig-zag model for laminated shells by assuming the in-plane displacements to be piece-wise linear through the thickness and the

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transverse displacement to be independent of the thickness coordinate. The continuity of transverse shear stresses at layer interfaces have been satisfied and the governing equation contains the same number of variables as in the FSDT. A similar theory has been developed by Librescu and Schmidt (1991) for geometrically non-linear shells. These theories produce uniform transverse shear stresses across the entire thickness of the shell and therefore compatibility conditions on the external bounding surfaces are not fulfilled.

Jing and Tzengt (1993) developed analytical solution for analysis of thick laminated cross-ply cylindrical shell panels using mixed formulation. The displacement field consists zigzag function in addition to the Reissner-Mindlin type inplane displacements and a constant transverse deflection. The effect of transverse shear deformation has been included through an independently assumed transverse shear stress field and the final governing equations contains seven unknowns. The initial curvature effect has been also incorporated in the strain-displacement relations, stress resultants and the assumed shear stress field. Xavier *et al.* (1993) proposed a variationally consistent higher order model for bending of laminated composite shells which provides a cubic variation of both the in-plane displacements and the shear stresses within each layer. The displacement model satisfies the zero transverse shear stress condition at the free surfaces. Since the continuity of in-plane displacement and shear stresses at the interface is imposed, the number of variable is same as those in FSDT, irrespective of the number of layer considered.

He (1994) obtained an exact unified representation of displacement variation through the thickness of a laminated shell by rigorous kinematical analysis. Then, an approximate theory of laminated shells has been established accounting for continuity of displacements and transverse shear stresses at layer interfaces. The governing equations contain only five unknowns. Di and Rothert (1995) presented an unconstrained third-order shear deformation theory for the analysis of laminated anisotropic cylindrical shells by considering a zig-zag function for the in-plane displacements. The zero-shearing condition on the laminate surfaces and continuous conditions for the transverse shear stresses on the inter-laminar surfaces have been considered for the final stresses calculation, the displacement functions remain to be unconstrained. This theory requires C^0 continuity for the assumed displacement fields for the finite element analysis. Shu (1996) developed analytical solution for laminated composite shallow shells which accounts

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for parabolic distribution of the transverse shear stresses through the thickness of the shell and the continuity of transverse shear stresses across each layer interface. This theory has been further extended to obtained closed-form solutions of simply-supported cross-ply both shallow and deep shells (see, e.g., Shu (1997)). The number of the displacement unknowns and the order of the equilibrium equations are the same as in the first-order shear deformation theory.Bhaskar and Varadan (1991) proposed a higher order zig-zag model with nine variables for bending analysis of laminated shell of revolution.

Cho and Parmerter (1993) proposed a theory for composite plates, where, in-plane displacement through the thickness is constructed by superimposing linear zig-zag field to the globally cubic varying field. The out-of-plane displacement has been assumed as a smooth parabolic variation through the thickness. Transverse normal strains are neglected. The layer dependent displacement variables have been eliminated by imposing transverse shear stress continuity conditions at the lamina interfaces and considering transverse shear stress free conditions of top and bottom surfaces of the laminate. Cho and Parmerter (1994) developed finite element models based on the above theory. The FE deal with the same number of degrees of freedom as the FSDT but require the C^1 continuous interpolation for the transverse deflection owing to the appearance of the second derivatives of the transverse deflection in the energy expressions. However, the construction of the C^1 continuous trial/test functions is not only computationally involved but also algebraically complicated. This theory is further extended by Cho *et al.* (1996) for symmetric laminated composite shells where the analytical solution for a cylindrical bending problem is obtained. Higher order cubic zig-zag theory for smart laminated thick composite cylindrical shell has been developed by Oh and Cho (2007).

To overcome a similar type of strict continuity requirement (i.e., C^1 continuity) placed by the conventional potential energy based variational formulation in the context of thin plates, several alternative formulations and the associated elements have been developed (see, Tsay and Reddy (1978) and Spilker (1980)). These include the hybrid and mixed finite elements which are mostly limited to CPT or FSDT. Theoretically, the mixed variational principles may be used to derive the discrete equations associated to the assumed independent expansions of the displacements and stresses. But these are seldom used in practice because of the difficulty in developing a reliable finite element model. This is one of the

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reasons behind the popularity of the displacement based finite elements, despite its shortcomings like poor prediction of the stresses compared to a hybrid or mixed finite elements. Putcha and Reddy (1986) are probably the first to develop a consistent and variationally correct C^0 type finite element based on the *third order shear deformation theory*(TSDT) considering a large number of nodal fields (i.e., 11 degrees of freedom per node or 44 degrees of freedom for a 4-noded element).

1.2.2 Coupled Piezo-Elastic Smart Composite Shells

The piezoelectric effects can be described as transfers between electrical and mechanical energy. The *direct piezoelectric effect* is the ability of certain crystalline materials to generate an electrical charge in proportion of an externally applied force, whereas in case of *inverse piezoelectric effect*, an electric field induces a deformation of the piezoelectric material. Mindlin (1952) is, probably, the first to develop the two-dimensional equations of motion of piezoelectric crystal plates from the three-dimensional classical equations of piezoelectricity. The equations involves only early terms of series expansions of the mechanical displacement and electric potential in powers of the thickness coordinate of the plate. Face-extension and face-shear modes were included by Mindlin (1961) in addition with flexure, thickness-shear and thickness-twist modes. Later Mindlin (1972) employed the variational principle to deduce the classical equations of low frequency vibrations of anisotropic plates from the three-dimensional equations of elasticity. Electric field and proper shear correction factor were considered in the formulation and the thickness modes were included in addition to the other modes assumed in his earlier work Mindlin (1961). The basic theory of linear piezoelectricity by Cady (1964), Tiersten (1969), Nye (1972) and Patron and Kudryavtsev (1988) are noteworthy. Early computational studies on this area have been carried out by Eer Nisse (1967a,b), Holland (1968) and Allik and Huges (1970).

1.2.2.1 Coupled Electro-Elastic ESL Models

In the early stage of the development of models, classical/first order shear deformation theory has been employed to predict the mechanical behavior of embedded or surface bonded piezo-electric layer. Lee (1990) was the first to present a multi-layered concept for piezoelectric plates and derived the governing equa-

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tions with reciprocal relationship based on the CLT. Saravanos (1997) developed coupled mixed theory for smart piezoelectric composite shell with the FSDT hypothesis and a LW approximation of the electrostatic potential along with the corresponding finite element for piezoelectric shells. Mitchell and Reddy (1995) proposed a hybrid plate model for smart composite laminates which consists of HSDT and LWT for the modeling of mechanical and electrical fields respectively. Pinto Correia *et al.* (1999) developed a semi-analytical axi-symmetric shell finite element model with embedded or surface bonded piezoelectric rings actuators or sensors for active damping vibration control of the structure. Pinto Correia *et al.* (2002) presented mixed finite element model based on higher order ESL theory to represent the mechanical behavior with a LW description in the thickness direction to represent the distribution of the electric potential of each piezoelectric layer of a frusta conical finite element. Finite element model based on FSDT description of displacement and LW form of the electric potential has been developed by Sheikh *et al.* (2001), Kulkarni (2003) for thin to moderately thick smart laminates. Similar approach has been proposed by Ballhause *et al.* (2005) for statics and dynamics of piezoelectric plates. Berg *et al.* (2004) presented a new equations of motion for the vibration of piezo-ceramics thin-walled cylindrical shells, generalizing Flugge's shell theory. The electric field has been obtained by solving an additional differential equation in the thickness direction. Analytical solution, based on a higher order shear and normal deformation theory, is presented for the cylindrical flexure of piezoelectric plates by Kant and Shiyekar (2008).

1.2.2.2 Coupled Electro-Elastic LW and Three Dimensional Model

Robbins and Reddy (1993) presented a layer-wise displacement theory and its FE model for piezoelectrically actuated beam. Layer-wise plate element with mechanical displacement and electric voltage as field variables has been proposed by Heyliger *et al.* (2003), Heyliger *et al.* (1996). Where each interface contains unknown for three displacement component along with the electric potential. The variation of these quantities across the plate thickness has been represented by piecewise linear interpolation functions along with the unknowns at the different interfaces. This model has been extended by Saravanos *et al.* (1997) for free vibration analysis. Han and Lee(1998) developed FE model for composite plates with distributed actuators based on LWT. However, the full LWT based formula-

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tions have been found to be computationally expensive particularly for a laminate with a large number of layers.

Exact solutions for simply-supported, laminated piezoelectric plates have been obtained by Ray *et al.* (1993), Heyliger (1994, 1997*a*), Heyliger and Saravanos (1995), Heyliger and Brooks (1996) and Xu and Tang (1995). Exact free vibration behavior in cylindrical bending has been studied by Heyliger and Brooks (1995) for orthotropic layers and Heyliger (1997*c*) for laminates containing off-axis orthotropic layers. Detailed review of the analytical solutions for laminated piezoelectric structures has been presented by Saravanos and Heyliger (1999). Kapuria *et al.* (1997) developed a three-dimensional solution for simply supported piezoelectric cylindrical shells subjected to axisymmetric electromechanical load. Analytical or exact solutions for laminated piezoelectric cylinder have been given by Mitchell and Reddy (1995), Xu and Noor (1996), Heyliger (1997*b*) and Dumir *et al.* (1997). Although these studies are useful but they concentrated on specific geometries and/or boundary conditions to analyze a specific type of behavior. Three-dimensional models are not computationally tractable for multi-layered smart shell structure.

Semi-analytical axisymmetric shell finite element model has been developed by Santos *et al.* (2008) for bending and free vibration of multilayered cylindrical shells with piezoelectric layers using the 3D linear elasticity theory. The equation of motion in the 3D axisymmetric model has been expressed by expanding the displacement field using Fourier series in the circumferential direction. Thus 3D elasticity equations of motion are reduced to 2D equations involving circumferential harmonics. In the finite element formulation the dependent variables, electric potential and loading have been expanded in truncated Fourier series. Dokmeci (1974) presented the higher order theory of piezoelectric shells in invariant form that contains both mechanical displacement and the electric potential of order (n) for dynamic study.

1.2.2.3 Coupled Electro-Elastic ESL Combined with ZZ Kinematics

Cho and Oh (2004) developed a higher order zig-zag plate theory to refine the prediction of fully coupled mechanical, thermal and electrical responses of smart composite plates. A three nodded triangular finite element has been developed by Oh and Cho (2004), based on higher order zig-zag plate theory to analyze the behavior of the composite plates with piezo-layers. Later, Oh and Cho (2007)

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extended the higher order zig-zag plate theory to develop a higher order zig-zag theory for smart laminated thick composite cylindrical shell. Where, in-plane displacement through the thickness is constructed by superimposing linear zig-zag field to the smooth globally cubic varying field. The out-of-plane displacement has been assumed as a smooth parabolic variation through the thickness. The layer dependent displacement variables have been eliminated by imposing transverse shear stress continuity conditions at the lamina interfaces and considering transverse shear stress free conditions of top and bottom surfaces of the laminate. Thus, the formulation involved displacement variable, which are layer independent. Degrees of freedom for electric potentials have kept layer-dependent in order to describe arbitrary distributions of electric potential through the thickness of smart structures.

1.3 Appraisal of the Past Work

A detailed review of the open literature on multilayered composite shells and the coupled electro-elastic field problem of smart composite shells, in the context of the present work brings out the following observations:

1. It has been observed that the ESL models often provide a sufficiently accurate description of the global response of thin to moderately thick laminated composite shells (e.g., transverse displacements, stress resultants, natural frequencies and associated mode shapes). Despite their inherent simplicity and low computational cost, the ESL models have several limitations that prevent them from being used to solve the whole spectrum of composite laminate problems. The accuracy of the global response predicted by the ESL models deteriorates as the laminate becomes thicker. Also, the ESL models are often incapable of accurately describing the state of stress and strain at the ply level near geometric and material discontinuities and they fail to satisfy the continuity conditions for the transverse shearing stresses at the lamina interfaces.

2. Layer-wise model can adequately describe the deformation behavior through the thickness of laminated composite shells but it is computationally expensive because it employs a large number of degrees of freedom which depends upon the number of layers.

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3. While the multi-layered plates based on the refined ESL combined with Zig-Zag(ZZ) kinematics (HSDTZZ) are many, the same applied to shells are rare. Here, a piecewise continuous displacement field (IC) in the thickness direction is generally assumed first and the interlaminar continuity of transverse shear stresses (ZZ) at each layer interfaces is imposed afterwards to obtain the compact HSDTZZ kinematics. Again, the HSDTZZ based elements deal with the same number of degrees of freedom as the FSDT. Most of the available works on laminated shells are based on the HSDT kinematics without the ZZ effect and do not fulfill the interfacial stress continuity conditions. However, the FE formulation based on the HSDT kinematics that satisfy the traction-free surface conditions require the C^1 continuous interpolation for the transverse deflection owing to the appearance of the second derivatives of the transverse deflection in the energy expressions.

4. The requirement of C^1 continuity in the FE formulation placed by the conventional potential energy based variational formulation in the context of plates have been circumvented by several alternative formulations. Most of which include the hybrid and mixed finite elements. However, they are mostly limited to CPT or FSDT.

5. Coupled electro-elastic plates and shells have been often modelled combining ESL theory for elastic displacements and LW description for electric potential. Most of the situations, the mechanical part has been assumed to have the HSDT kinematics and the electrical part has been assumed to follow a piece-wise linear LW distribution. However, the full LWT based formulations have been found to be numerically inefficient particularly for a laminate with a large number of layers.

6. Coupled electro-elastic laminated plates/shells combined with the HSDTZZ kinematics for the mechanical part and the higher-order LW distribution for the electric potential are rare. Moreover, the finite elements in these regime are rarely available.

1.4 Objectives and Scope of the Present Work

Based on the review of open literature and critical observations as above, the objectives and scope of the present research are accordingly defined as follows:

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The objective of the present research work is to develop an elegant higher order shear deformable coupled electro-elastic general shell model that accounts for the continuous interlaminar transverse shear stresses and the transverse electric displacement, including the mixed finite element implementation.

In other words, the objective comprises the following features:

- Development of a higher order ESL type electro-elastic general shell model that involve minimal number of unknowns.
- The transverse shearing stresses should be continuous at the interfaces and must get vanished at the top and bottom surfaces of the shells.
- The transverse electric displacement vector should be continuous at the interface and the charge boundary conditions at the top and bottom surfaces of the shells should be satisfied.
- Construction of a suitable mixed functional of modified *Hellinger-Reissner* type that involve only the first derivatives of all the independent fields via considering the moment stress-resultants as primary unknowns (C^0 continuity requirement). Moreover, consideration of moments as primary variables will offer the advantage of satisfying the traction boundary conditions easily.
- Unlike the various shell theories, in which the cross-section rotations are considered as the independent field, here the average cross-section shearing strains ($\overset{0}{\gamma}_{1z}, \overset{0}{\gamma}_{2z}$) will be treated as the independent field. Such type of parametrization may be a suitable alternative to make the kinematics simple.
- The finite elements should be based on mixed-variational principle and are to be free from shear and membrane lockings. More importantly, these elements must be free from any spurious modes.
- Employment of polynomial multi-scaling for the electro-elastic shell kinematics (i.e., different polynomial degree for the mechanical and the electric potential parts) that offer to treat the interfacial electric boundary conditions in a weak sense.

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- The development of a simple least-square FE in the thickness direction for the accurate estimation of the transverse stresses. As the finite elements based on the higher order shear deformable equivalent plate theory coupled with a simple post-processing stage for the accurate estimation of the transverse stresses may be a better alterative.

While the objectives have been kept quite general in nature, the scope of the present investigation has been decided to consider the theoretical problems associated to the followings:

- Bending analysis of coupled electro-elastic doubly curved shells. Static behavior of spherical, cylindrical, hyperbolic paraboloid and hyper shells subjected to

  1. Mechanical load
  2. Electric load
  3. Complex electro-mechanical load

  Accurate estimation of both the transverse mechanical stresses and the electric displacement vector under different lamination scheme is proposed. Influence of mechanical and electric boundary conditions, length to thickness ratio (a/h) and curvature to length ratio (R/a) on the mechanical deflection, voltage, electric displacement and mechanical stresses for different doubly curved shells are studied.

- Coupled free vibration analysis of electro-elastic doubly curved shells is analyzed. Influence of boundary conditions, length to thickness ratio (a/h) and curvature to length ratio (R/a) on the natural frequencies, mode shapes for different doubly curved shells are investigated.

The fundamental equations of piezoelectricity for laminated composite shells have been derived. The resulting FE equations have been developed using mixed variational principle. The FE codes are developed using FORTRAN 90 for coupled electro-elastic laminated spherical, cylindrical, hyperbolic paraboloid and hyper shells, where provisions are kept for both the 4-noded as well as the 9-noded quadrilateral elements. The developed FE codes have been used to generate results for specific plate and shell problems. Finally, important conclusions are drawn based on the analysis of the present results. The scope for future research in the appropriate direction is also identified.

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1.5 Organisation of the Thesis

The research works that have been carried out under the proposed title are presented in six chapters. A description of each chapter is briefly outlined below:

Chapter 1 contains a brief introduction of laminated composite shells and smart shell with main emphasis on modelling. A brief review of the open literature regarding the modelling of laminated composite shells and piezo-elastic laminated composite shells is added subsequently. Based on the review of the related literatures and critical observations the objective and scope of the present work have been defined.

Chapter 2 deals with the basic assumptions and necessary theoretical development of the governing equations for doubly curved piezoelectric laminated shells considering interlaminar shearing stress continuity along with the boundary conditions. Construction of a suitable mixed functional of modified *Hellinger-Reissner* type that involve only the first derivatives of all the independent fields via considering the moment stress-resultants as primary unknowns (C^0 continuity requirement) is presented. The finite element formulations and detailed implementation procedures are presented subsequently.

Chapter 3 is related to the linear bending of composite and sandwich shells without any embedded or surface bonded piezoelectric layers subjected to mechanical load. The obtained results are compared with the available close form and analytical results from the open literature for plates and shells. New results are presented in tabular and graphical form for the mechanical deflection and stresses under different lamination scheme. Influence of geometric boundary conditions, length to thickness ratio (a/h) and curvature to length ratio (R/a) on the mechanical deflection and stresses for spherical, cylindrical, hyperbolic paraboloid and hyper shells are studied.

Chapter 4 is related to the bending of coupled electro-elastic shells subjected to mechanical, electrical and complex electro-mechanical load. The obtained results are validated with the available close form and analytical results from the open literature for plates. New results are presented in tabular and graphical form for the mechanical deflection, mechanical stresses, voltage and electric displacement vector under different lamination scheme. Influence of geometric and electric boundary conditions, length to thickness ratio (a/h) and curvature to length ratio (R/a) on the mechanical deflection, voltage, electric displacement and mechanical

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stresses for different doubly curved shells are studied.

Chapter 5 deals with the coupled free vibration analysis of electro-elastic doubly curved shells. Natural frequencies under different lamination scheme are computed. Influence of boundary conditions, length to thickness ratio (a/h) and curvature to length ratio (R/a) on the natural frequencies, mode shapes for different doubly curved shells are investigated.

Chapter 6 contains the critical discussion and important observations on the numerical results obtained in the previous chapters and concluding remarks on the work carried out in the present thesis. This is followed by a brief outline of the scope of the further investigations in this research area.

A list of the references has been appended at the end.

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