Chapter - 1

INTRODUCTION

1.1 Introduction

Plates used as structural elements take different shapes due to their functional or structural requirements as well as from the aesthetic consideration. These arbitrarily shaped, elastic thin plates are widely used in civil, marine, aeronautical and mechanical engineering applications. Various engineering structures consisting of these thin plates of different shapes are often stiffened with stiffening ribs for achieving greater strength with relatively less material and thus making the structure cost effective. While the stiffening elements add negligible weight to the overall structure, their influence on strength and stability is enormous. In this process the strength/weight ratio is improved dramatically which is vital in some specific structures like ship, aircrafts and similar other types.

These plates of arbitrary geometries are subjected to the static lateral load, the dynamic load and the inplane load for which three types of analysis such as static, free vibration and stability are to be carried out. In these analyses, the geometry of the plate as well as its boundary conditions play a major role in the choice of the methods of the solution. Exact solutions for plates are available only for certain shapes, boundaries and loading conditions. An attempt to have an analytical solution of the arbitrarily shaped plates with complex boundary conditions may lead to an extremely tedious though not impossible task because of the complex nature of the problem arising out of a curved boundary. As a result, various methods such as Rayleigh-Ritz method, Galerkin method and the likes have been used by several investigators depending on the suitability of the problem. Some investigators [83] have attempted the conformal mapping [166] for solving plates of regular polygonal shape whereas some have used the finite strip method [34] and the spline finite strip method [37] for solving problems relating to plates of arbitrary geometry apart from the popular finite element method. Investigators from various fields have contributed to the study of bare plates and stiffened plates making the library of literature rich in the area of static, dynamic and stability analyses of these plates.

The stiffened plates consist of a skin and a varying number of ribs. The skin is termed as *plate* throughout this thesis and the terms such as rib, stiffener, girder, beam and stringer are used interchangeably to indicate the ribs. When the rib centroid is coincident with the plate middle surface, no inplane stresses are developed due to the bending of the stiffener and this class of stiffened plates is identified as *concentrically stiffened plates*. In the other case, when the rib centroid and the plate middle surface are eccentric, the inplane stresses developed in the plate due to the stiffener bending have to be considered and this class of stiffened plates is designated as *eccentrically stiffened plates*.

The optimum design of stiffened plate structures demands an effective analytical procedure. But the stiffening arrangements pose another difficulty in addition to the inherent problem due to the diverse geometrical configurations, loading and boundary conditions encountered in case of bare plates for obtaining a suitable theoretical solution. Hence, the earlier investigators modelled the stiffened plated system into a simpler structural form such as an orthotropic plate or a grid system which were amenable to the solution procedure developed at that time. As the orthotropic plate model is applicable to closely equispaced stiffeners of equal size only and the grid model can perform well only in case of orthogonal stiffeners, the applicability of the models to a generalized problem is severely restricted because of the simplicity inherent in the approximations.

The emergence of the digital computers with their enormous computing speed and core memory capacity has changed the outlook of the structural analysts and caused the evolution of various numerical methods such as the finite element, the finite difference, the finite strip and the boundary element method. These numerical tools allow the researchers to model the structure in a more realistic manner with simpler mathematical forms.

In view of the availability of the computational facility, the orthotropic and the grillage models can be replaced by the plate beam idealization where the plate and the stiffeners can be modelled separately maintaining the monolithic connection between the two and then one of the numerical methods may be applied for their analysis. Among all the numerical methods, the finite element method has been found to be a powerful, versatile and accurate one in the analysis of complex structures. But, in the finite element analysis of plates with arbitrary configurations, the main problem arises in the choice of a suitable element, as many of the present elements are unable to cater to the arbitrary plate geometry.

In the past the most common approach to the finite element analysis of arbitrarily shaped plates has been to approximate the curved boundaries with a large number of straight-edged triangular elements [7] [9] [63] [108] or developing special purpose elements permitting the exact representation of curved boundaries [141] or using a triangular element with one of the sides being modified to include a curved edge [35]. But these elements being developed to accommodate a particular plate geometry, none of them can be generalized to represent an arbitrary edge such as straight, skew or curved.

Another successful approach in this pursuit is the application of the isoparametric element because of its generality to model a curved boundary successfully. Unfortunately, this element which is having the shear strain term based on the Mindlin's theory becomes very stiff when used to model thin structures, resulting inexact solutions. This effect is termed as shear-locking which makes this otherwise successful element unsuitable. Much effort has been put to identify and eliminate the source of this shear-locking effect. The most successful technique for alleviating the problem associated with this shear-locking is through evaluating certain transverse shear coefficients of the element stiffness matrix using a lower order numerical integration rule than that which is required to evaluate the coefficients exactly as discussed by Zienkiewicz and Taylor [199]. This technique which is known as reduced or selective integration has been used on elements which shear-lock when exact integration is performed. However, an inexact integration scheme results in a rank deficient element stiffness matrix, which in turn, generates additional zero strain deformation modes in a solution known as zero-energy modes, other than the rigid body movements and which must be suppressed through stabilization techniques. It has been found that all the displacement-based shear deformable plate elements of this kind fail on many occasions either by shear-locking or singular behaviour.

Thus it is felt that in spite of vast number of elements present in the literature [67] since the inception of the finite element method in the early 1960s, still there is a need of development of suitable elements which can model the thin plates of arbitrary geometry successfully.

1.2 The Objective and Scope of Present Investigation

The objective of the present investigation is to formulate simple and efficient finite elements for static, free vibration and buckling analyses of the bare and stiffened plates of arbitrary geometrical shapes under diverse loading and boundary conditions and demonstrate the performance of the proposed elements through the numerical examples in the related fields.

In this thesis, a new four-noded plate bending element is proposed for the analysis of the bare plates, which is derived, though from the simplest rectangular basic plate bending element having 12 degrees of freedom largely known as *ACM Element* [1], but it has all the advantages of the isoparametric element to model an arbitrary plate shape and without the disadvantage of the shear locking problem. Further, for the analysis of the stiffened plates, a stiffened plate bending element is formulated by combining the four-noded rectangular plane stress element having 8 degrees of freedom with the 12 degrees of freedom *ACM Plate Bending Element*. The incorporation of boundary conditions is made in the most general manner to cater to the need of the curved boundary as well as to the more practical mixed boundary conditions.

As the element developed for the bare plate analysis is capable of modelling an arbitrary plate geometry, a large number of static, dynamic and stability problems in the bare plate domain of square, rectangular, skew, trapezoidal, triangular, circular, elliptical, annular sector geometries are considered and the results are presented showing the elegance and efficiency of the proposed element.

The element developed for the analysis of the stiffened plates has the same feature of accommodating the arbitrary shape of the plate geometries and the stiffener modelling is done for a general one. The stiffener is modelled in such a way as to lie anywhere within the plate element and need not follow the nodal lines. Further, in the formulation, their orientation is kept arbitrary which makes the analysis more flexible and the mesh division independent of their location and orientation. The same displacement interpolation functions as used for the plate elements are adopted in the formulation of the stiffener element. This facilitates to express the stiffness and the mass matrices of the stiffener in terms of the nodal parameters of the plate element thus ensuring the compatibility of the stiffener with the plate.

Similar to the bare plate; static, dynamic and stability analyses of various stiffened plate configurations such as square, rectangular, skew, trapezoidal, triangular, circular, elliptical, annular sector etc. with various stiffener positions have been carried out.

The implementation of the methodology to different types of analysis described in the investigation is made through the development of computer programmes in **C++**. To make the analysis more cost effective, the global elastic stiffness, mass and geometric stiffness matrices are stored using the skyline storage technique. No standard or general software package is used for these analyses and as such the computer programmes developed here are general and complete in themselves. The computer programmes have been run in the **HP - UX 9000/819** work station available at the Computer Centre of the Institute and the **ORIGIN 200** of the Departmental Computer Laboratory.

The present investigation comprises the following topics:

1. **Analysis of Arbitrary Bare Plates**

   (a) **Static Analysis of Arbitrary Bare Plates :** The static analysis is carried out for different geometrical plate shapes such as square, rectangular, skew, annular sector and circular one for various boundary and loading conditions and the results are compared with the published ones.

(b) **Free Vibration Analysis of Arbitrary Bare Plates :** The proposed element is tested by considering the free flexural vibration analysis of bare plates of various shapes having various boundary conditions and the first few natural frequencies are compared with those from open literature.

(c) **Stability Analysis of Arbitrary Bare Plates :** In the stability analysis, bare plates of rectangular, skew and circular configurations with different boundary and inplane loading conditions are considered and the results are validated by comparing the buckling parameters obtained with those available ones.

2. **Analysis of Arbitrary Stiffened Plates**

(a) **Static Analysis of Arbitrary Stiffened Plates :** A large number of stiffened plates of straight and curved edges with concentric as well as eccentric stiffeners are studied. The results are presented in terms of stresses/stress resultants. Some new results are also presented.

(b) **Free Vibration Analysis of Arbitrary Stiffened Plates :** The first few natural frequencies of a large number of stiffened plates having various planforms are presented. In the analysis, eccentric as well as concentric stiffeners are considered and various boundary conditions are implemented. Some new examples are also attempted.

(c) **Stability Analysis of Arbitrary Stiffened Plates :** Stability analysis is carried out for rectangular, skew and circular stiffened plates with various boundary conditions and buckling parameters are presented for various flexural and torsional stiffness of the stiffeners. Few new results have been presented for this category of analysis.

Hence, in summary, a large number of numerical examples have been considered in this investigation for static, dynamic and stability analyses of bare plates and stiffened plates of various geometrical configurations. Various loading and boundary conditions as well as concentric and eccentric stiffeners are considered in the analysis. In addition to the examples presented for the validation of the proposed method some new results are also put forward.