CHAPTER I

# GENERAL INTRODUCTION

The fixed point theory, as it stands today, is not only a branch of functional analysis, but is a subject in its own right. During last two decades the progress of fixed point theory is remarkable. It has not only gained its strength, it has nurtured others with tender care. With the help of this theory it has been possible today to treat many apparently diverse problems of analysis and algebra from the same platform and as such it has provided a unifying approach to these problems. This theory has been effectively exploited in classical analysis, numerical analysis, algebra, differential and integral equations, linear programming and social sciences. There appears to be a promise in further applications.

Two earlier theorems, one due to Banach (1922) and the other due to Schauder (1930), have laid the foundation of fixed point theory. Both the theorems have vast applicability, but Banach's theorem appears to have drawn more attention of researchers in this field and its generalizations run into thousands of pages of literature. Banach's theorem, better known as Banach's contraction principle, establishes the existence and uniqueness of fixed points of contraction mappings. A minor shortcoming of this theorem and its generalizations derived till 1968 is that the mappings involved are necessarily continuous.

Kannan (1968, 1969) first derived a generalization of Banach's theorem, where the mapping, though contractive type, need not be continuous. Since then much attention has been focussed on this aspect and the present thesis is the outcome of an attempt in this direction.

The thesis is divided into eight chapters including the present one.

Chapter II deals with strict generalized contraction principle obtained by extending the principle of generalized contraction introduced by Ćirić (1971). The existence and uniqueness of fixed points of strict generalized contraction mappings have been discussed by Rhoades (1977). But it has been established here that the results of Rhoades may be derived under some weaker conditions. Indeed, the continuity of the mapping and some of its iterate has been assumed at the fixed point, whereas the theorems of Rhoades demand the continuity of the mapping throughout the space over which the mapping is defined. Fixed point theorems have been derived considering the mappings singly and in pairs. The results of this chapter form the basis of a publication in Matematički Vesnik <u>13</u> (28) (1976) by the author.

Chapter III is devoted to a discussion on weakly uniformly strict generalized contractions. Meir and Keeler (1969) have extended Banach's contraction principle by introducing weakly uniformly strict contractions. Following Meir and Keeler the Ćirić's principle of generalized contractions has been extended in this chapter and thereby fixed point theorems have been established considering them singly and in pairs.

Chapter IV deals with fixed point theorems extending the principle of non-linear contractions of Boyd and Wong (1969). These extensions have been derived in the same way as Sehgal (1972) has extended the work of Edelstein (1962).

The results of Chapters III and IV are based on author's published work in Publications de L' Institut Mathematique, 21 (35) (1977), 115-118.

Chapters V and VI deal with orbital contractions and their fixed points. Here the mappings are contractive only over two consecutive elements of an orbit. It has been shown that the results of this chapter are advantageous over many others in fixed point theory. The principle of orbital contractions has been extended further by Rhoades (1978). The results of this chapter have been published by the author in Proceedings of the American Mathematical Society, 64 (1977), 283-286, Glasnik Matematički, 13 (33) (1978), 81-90 and Matematički Vesnik 3 (16) (31) 1979, 31-32.

Chapters VII and VIII are devoted to discussions on fixed point theorems obtained by extending the principle of quasi-contractions of Ćirić (1974). It has been shown in Chapter VII how the conditions can be weakened in the principle of quasi-contractions. In Chapter VIII orbital quasi-contractions have been derived as an extension of quasi-contractions and thereby fixed point theorems have been established both for a single mapping and a pair of mappings. The results of Chapter VIII form the basis of a publication by the author in Mathematica Balkanica, 6(26) 1976, 152-154 and Matematički Vesnik, 1(14) (29) 1977, 285-287.

Examples are considered in each chapter for illustrations and to show further that the results of the thesis are indeed extensions.