

SYNOPSIS

Biomechanics is that branch of science, which deals with the application of the principles of mechanics on living organisms. It also seeks to understand in an accurate manner the behavioural details of various biological systems through successful applications of the said principles on physiology and medicine. The growth of the subject was initiated on the basis that functioning of different physiological organs may be better understood when the knowledge of already developed branches of science like fluid mechanics, solid mechanics, gas dynamics etc. are incorporated. It is now a well understood fact that physiology can be better understood with the aid of Biomechanics.

An ill health may be regarded as the absence of normal functioning of certain physiological organs. One example of this is an aneurysm in blood vessels which is commonly known as stenosis. In case of such a disease, flow pattern of blood is deviated from its normal course and the pressure field is greatly disturbed. The obvious consequence of this is a disease known as thrombosis. With the help of fluid mechanical aspects, in recent years attempts are being made to study the flow field in the stenosed region of blood vessels. This also suggests a possible method of remedy in the sense that the informations derived on the basis of relevant investigations are useful to the clinicians for the treatment of the aforementioned vascular disease.

In order to understand the dynamical behaviour of the cardiovascular system, in particular the circulatory system, a study of the flow of blood through different blood vessels is quite useful. The flow of blood in the circulatory system is initiated by the contraction of the dynamic organ, the heart, which pumps out blood into the distensible aorta and the pulmonary arteries by way of transmission of pressure pulses into them. The intermittent pumping of the heart is converted into a smooth flow of blood in the blood vessels. The circulation of blood back into the heart is a necessary condition for the normal functioning of the heart.

One of the primary purposes of Biomechanics is to increase the fundamental knowledge about living organisms and any artificial intervention, e.g. designing of artificial internal organs like artificial kidney, artificial heart, heart-lung machine and renal replacement. One of the recent fields of interest in Biomechanics is concerned with investigations of human tolerance to vibration environments, acceleration and gravitational variation. On the other hand, instrumental application of Biomechanics lies in the designing of advanced instruments such as electromyograms, electrocardiograms etc. to measure a variety of physiological parameters like blood pressure and heart beats.

It is a well established fact that the advancement of science and technology owes a great deal to the development of mathematical methods and models. Mathematical models are

now-a-days being successfully developed in order to study various branches of science, including physiology, applied psychology and medicines.

Some of the previous investigators have studied mathematical models on blood flow through different parts of the cardiovascular system analytically as well as numerically. Their models have provided a variety of useful informations regarding different diagnostic tools, have suggested guidelines for prosthesis and above all, have put forward enormous numerical data having paramount importance.

The present thesis is concerned with the development and analysis of several mathematical models in connection with the study of different aspects of the problem of blood flow through various parts of the cardiovascular system, under different physiological conditions, by employing the valid principles of fluid mechanics. Basically it consists of several analytical investigations on the flow behaviour of blood in different blood vessels in pathological as well as normal states. The thesis contains seven chapters. The first chapter is introductory. It presents a brief discussion on human physiology, applications of mechanics in physiology, cardiovascular system, blood circulation and the circulatory system, constitutive equations for blood, rheological properties of blood, equation of motion applicable to blood flow, stenosis and a brief account of the relevant previous investigations.

The second chapter is devoted to a study of a boundary layer formation in the steady flow of blood. In this chapter, blood is treated as an incompressible Newtonian fluid. Considering the flow of blood through an entry section, an attempt is made to investigate the formation of boundary layer in a blood vessel of the cardiovascular system. Consideration is made of a top-flat velocity profile which combines the potential flow with the boundary layer. When blood enters the vessel, the velocity profile is taken to be initially flat. As it proceeds along the vessel, the velocity profile continually changes to the top-flat profile. The usual concept of boundary layer is introduced in the sense that a layer exists towards the boundary of the vessel on which the shear stress is equal to the yield stress of blood. Expressions for the skin-friction and pressure in the entry region are derived. To make the theory more meaningful, the concept of displacement thickness is introduced.

The third chapter deals with a study of the flow field of blood through small arteries of the circulatory system under the influence of a vibration environment. The effect of micro-rotation of the blood cells has been duly accounted for. The momentum integral method has been employed for studying the fully developed flow-field in arteries. Some specific situations of particular interest are analysed in a greater detail. These are : (i) steady flow with vanishing pressure gradient, (ii) steady flow with uniform pressure gradient and (iii) pure sinusoidal pressure gradient and wall vibration. The validity of the analysis is illustrated through computational results

for a specific artery subjected to longitudinal vibration.

The fourth chapter is devoted to an investigation of the flow behaviour of blood through the segment of an artery having a collar like mild stenosis. Two different models of blood flow are analysed, viz. the Bingham plastic fluid model and the Casson fluid model. In the first part of the analysis, the influence of the fluid model parameters representing the yield stress of blood and the Bingham viscosity, on the flow behaviour of blood is investigated. It is found that the resistance to flow and the skin-friction increase as the stenosis grows more and more. In the second part of the analysis, the Casson fluid model is examined. The resistance to flow and the skin-friction are also computed for this model. Significant differences in these values are noticed when compared to the first model. This observation emphasises the importance of paying due attention to the rheological properties of blood.

In the fifth chapter, a mathematical model is developed for the study of blood flow through a mammalian blood vessel in the presence of a stenosis by taking into consideration the velocity slip at the wall. By employing the momentum integral technique, analytical expressions for the velocity profile, pressure gradient and skin-friction are derived. The condition for the adverse pressure gradient is also deduced. It is observed that the slip velocity bears the potential to influence the velocity distribution of blood to a remarkable extent and to ^{influ} ~~the~~

considerably *the pressure gradient as well as* ~~the pressure gradient as well as~~ the skin-friction.

The sixth chapter deals with the development of a mathematical model with the purpose of studying the influence of plasma skimming on hematocrit reduction when blood flows from a cylindrical feeding tube into a cylindrical capillary at a right-angle branch. A two-fluid model for blood is considered. By accounting for the suspension of erythrocytes in the core region, the central layer is treated as a micro-polar fluid, while the peripheral plasma layer is treated as a Newtonian fluid. In order to consider the non-uniformity of the concentration of erythrocytes, two different concentration profiles, viz. (a) an abruptly varying concentration profile and (b) an almost linear concentration profile, have been examined with particular emphasis. Analytical expressions and numerical values are obtained for hematocrit reduction in the branch capillary.

Of concern in the seventh and the concluding chapter is an analysis of the Herschel-Bulkley fluid model for blood in the study of branching effects in the microcirculatory system. The problem considered here is similar to that analysed in the preceding chapter. The flow of blood is investigated by considering blood to be plastic of Herschel - Bulkley type. Red cells in blood are taken to be concentrated in the core region,



where the concentration profile is specified by a piecewise linear function. Based upon the said model, analytical expression for hematocrit reduction in the branch capillary is derived. By considering a specific numerical example, quantitative estimates of the concerned quantities are also obtained.