Abstract

The work presented in this thesis is devoted to the study of geodesic focusing and energy conditions in general relativity and modified theories of gravity. Using the well-known Raychaudhuri equations, one can show how geometric features of a given spacetime as well as shear and rotation, governs the *focusing or defocusing* of geodesic congruences. If rotation is set to zero, the *convergence condition* (timelike or null) emerges as a geometric condition for focusing.

Physical requirements on matter stress-energy (eg. positivity of energy density in any frame of reference), lead to the various *energy conditions* as independent inequalities, which must hold, at least in a classical context. The convergence conditions are related to the energy conditions in different ways (for general relativity/modified gravity), through the field equations of a given theory. In general relativistic spacetimes, a violation/satisfaction of the convergence conditions for geodesic congruences directly leads to a violation/satisfaction of the energy conditions on matter. In contrast, in certain spacetimes which arise as solutions in some modified theories of gravity, violation of the convergence condition on geodesic congruences may be possible with matter satisfying the energy conditions. The threads that link the convergence conditions, shear and rotation, energy conditions and focusing constitute the central theme of this thesis.

To this end, we first provide a starter by looking at families of trajectories and the analog of focusing in *classical mechanics*. We then move on to studying focusing issues in a known general relativistic, non-static spacetime modeling gravitational collapse of a massless scalar field. Thereafter, we shift to modified gravity and choose to look at *wormhole spacetimes* which are known to require exotic (energy condition violating) matter in the context of general relativity. We show that in two different *modified theories of gravity*– Eddington-inspired Born-Infeld gravity and an on-brane scalar-tensor theory – there exist wormholes with matter satisfying the energy conditions though the wormhole geometries necessarily violate the convergence conditions. Finally, we explore a possible link between *gravitational lensing* and the energy conditions, in brief. The thesis ends with a summary of the main results and the scope of future investigations.

Keywords: general relativity, geodesic focusing, classical mechanics, energy conditions, convergence conditions, modified gravity, wormholes, gravitational lensing.