Chapter 1

General Introduction

1.1 Introduction

There exists in the literature some theories viz., theory of probability, theory of fuzzy sets, interval mathematics, intuitionistic fuzzy sets, rough sets which have been widely used by various researchers to solve complicated problems involving uncertainties typical of those problems. We may recall that these theories were developed because the classical methods could not be successfully employed to solve such problems. All these theories have their inherent difficulties which have been discussed later on. Molodtsov [37] proposed a new theory named as 'Soft Set Theory' in 1999. This theory may be regarded as another major mathematical approach or tool for dealing with uncertainties. We believe that the theory of soft set is methodologically significant to deal with problems in the field of neural networks, soft computing etc. specially in the representation of and reasoning with vague and/or imprecise knowledge data analysis, machine learning etc. As a first step towards a further development of this new theory we have in this thesis proposed some algebraic definitions and results. Some examples related to decision making problems have been worked out as application of these results proposed by us.

1.1.1 Soft Set

As one understands uncertainty is a situational property of phenomena which has various causes and which is also influenced by available and required informations. Most of the established theories, viz., the probability theory, fuzzy set theory, rough set theory, theory of interval mathematics etc. for dealing with uncertainty modelling are focused either on specific types of uncertainty defined by their causes or they at least imply certain causes. They may also require particular types of information depending on the type of information processing they use. The above named mathematical tools in solving problems cannot be used successfully because of various uncertainties typical of these problems.

The soft set theory proposed by Molodtsov [37] may be regarded as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. Before we present the soft set theory proposed by Molodtsov it will not be out of the place to first briefly present the concepts of probability theory, fuzzy set theory, rough set theory with emphasize the difficulties inherent in these theories. We also briefly discuss the notion of intuitionistic fuzzy set which we have used in formulating intuitionistic fuzzy soft sets in chapter 4. Major research work carried out in these fields that are centered around the problem investigated by us have also been reported.

Probability theory

As is well known the theory of probability can deal only with stochastically stable phenomena. One can say that for a stochastically stable phenomena there should exists a limit of the sample mean μ_n is defined by

$$\mu_n = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

where, x_i =1 if the phenomena occurs in the trial and $x_i = 0$ if the phenomena does not occur. To test the existence of the limit we must perform for a large number of trials. However, though it is possible to carry out such performance in engineering related problems, we cannot expect to carry out the same when we have to deal with problems in other fields of interest viz., economics, environmental and social sciences.

1.1.2 Fuzzy Sets

With the advancement of science and technology our modern society becomes very complex and with this the decision process become increasingly very vague and hard to analyze. The human brain with its characteristics is able to learn and reason in a vague and fuzzy environment. One is able to arrive at decisions based on imprecise, quantitative data in contrast to formal mathematics and formal logic which demand precise and quantitative data. Modern computers possess capacity but lack this human like ability. Undoubtedly, in many areas of cognition, human intelligence far excels the computer 'intelligence' of today and the development of the fuzzy concepts is a step forward towards the development of tools capable of handling humanistic type of problems.

We do have sufficient mathematical tools and computer-based technology for analysing and solving the problems embodied in deterministic and uncertain (probabilistic) environment. Here uncertainty may arise from the probabilistic behaviour of certain physical phenomena in mechanistic systems. Though we are aware of the important role that vagueness and inexactitude play in human decision making, we did not know, until 1965, how the vagueness arising from subjectivity which is inherent in human thought processes, can be modelled and analysed.

In 1965, professor Lotfi A. Zadeh [66], now popularly known as the father of fuzzy logic, introduced the concept of fuzzy sets as a new way of representing vagueness in everyday life. This theory provides an approximate and effective means for describing the characteristics of a system which is either too complex or is ill-defined to admit precise mathematical analysis [67, 68, 69, 70]. Essentially the approach is based on the idea that the key elements in human thinking are not just the numbers but can be approximated to classes of objects in which the transition from membership to nonmembership is gradual rather than abrupt. Much of logic behind human reasoning is not the traditional two-valued or even multivalued logic, but logic with fuzzy truths, fuzzy connectives, and fuzzy rules of inference.

Fuzzy set theory is known to handle to a 'reasonable extent' uncertainties in various problems corresponding to decision making models under different kinds of risks, subjective judgement, vagueness, and ambiguity. There may however deficiencies due to incompleteness, impreciseness and vagueness depending on the problem. The theory looked upon as a generalization of the classical set theory, it has greater flexibility to capture various aspects of incompleteness or imperfection in information about a situation. Zadeh has developed a theory of approximate reasoning based on fuzzy set theory. Approximate reasoning refers to a type of reasoning that is neither very exact nor very inexact. This theory aims at modelling the human reasoning and thinking process with linguistic variables introduced by Zadeh [67, 68, 69], in order to handle various types of uncertainty. Many aspects of the underlying concept have been incorporated in designing decision-making systems [27, 74].

In the following we give a brief presentation of the mathematical representation of fuzzy sets.

A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Individuals, may belong to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades are very often represented by real number values ranging in the closed interval [0, 1]. Thus, a fuzzy set representing the concept of a sunny day might assign a degree of membership of 1 to a cloud cover of 0%, 0.8 to a cloud cover of 20% etc. These grades signify the degree to which each percentage of cloud cover approximates our subjective concept of sunny and the set itself models the semantic flexibility inherent in such a common linguistic term. Because full membership and full non membership in the fuzzy set can still be indicated by the values of 1 and 0, respectively, we can consider the concept of a crisp set to be a restricted case of the more general concept of a fuzzy set for which only these two grades of membership are allowed.

Thus a fuzzy set may be treated as a class that admits the possibility of partial membership in it. Each membership function maps elements of a given universal set X, into real numbers in [0, 1]. The membership function of a fuzzy set A is denoted by μ_A; that is

$\mu_A : X \rightarrow [0, 1]$.

For x ∈ X, μ_A(x) is called the grade of membership of x in A. When the membership value μ_A(x) is more close to 1, the element x will be more close to full membership of A. The grades 1 and 0 represents the full membership and non-membership respectively. The fuzzy set A is completely characterized by the set of pairs { (x, $\mu_A(x)$): x ∈ X }. The standard complement $\bar{A}$ of a fuzzy set A with respect to universal set X is defined by $\mu_{\bar{A}}$ (x) = 1 - μ_A(x).

For fuzzy sets A and B in X, we have

A = B iff $\mu_A(x) = \mu_B$ (x),

A ⊆ B iff $\mu_A(x) \leq \mu_B$ (x), and

A ⊇ B iff A ⊆ B.

The union A ∪ B can be defined by one of the following membership functions:

$\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$,

$\mu_{A\cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)$, and

$\mu_{A\cup B}(x) = \min\{1, \mu_A(x), \mu_B(x)\}$.

There are three possibilities of membership functions for the intersection A ∩ B viz.,

$\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$

$\mu_{A\cap B}(x) = \mu_A(x).\mu_B(x)$,
$\mu_{A\cap B}(x) = \max\{0, \mu_A(x) + \mu_B(x) - 1\}$.

Presently the theory of fuzzy set is progressing very rapidly and is being used in various decision making problems. To name a few we may refer to [11, 15, 16, 52, 71, 72, 73]. However as have been mentioned in the beginning of this chapter there are certain inherent difficulties in this theory. The difficulty is: how to set the membership function in each particular case? One should not impose only one way to set the membership function. The nature of the membership function is extremely individual. Such functions are to be constructed on the basis of situation or environment under study and it is subjective in nature. So, it cannot be assigned arbitrarily. In fact, all type of possible problems in our real life environment have not been yet classified into a finite (or, at worst, infinite) number of classes. The fuzzy set operations based on the arithmetic operations with membership function do not look natural. It may appear that the operations are similar to the addition of weights and lengths. The inadequacy of the parametrization tool of this theory possibly is the reason for these difficulties. As discussed later on we may note that the concept of soft set theory proposed by Molodtsov, is free from these difficulties.

1.1.3 Intuitionistic Fuzzy Sets (IFS)

Whenever a fuzzy-problem is under consideration the membership function (or membership values) are to be just somehow evaluated. The term, "evaluation" is important here. For, example, consider the fuzzy set, 'collection of all good students in a class X'. Here membership values corresponding to the qualification of goodness are to be evaluated to have a final structure of a fuzzy set. For every evaluation of such type, there is again a serious problem, "How to evaluate". Whether evaluation of some object can be done completely, i.e., with complete satisfaction ? Whether the evaluator is knowledgeable enough to evaluate an object completely ? As for example to justify the quality of a newly constructed building, some of the components lead to decide its 'goodness' and some certainly lead to decide the quality 'not-goodness'. But, there are some components that cannot lead to decide its 'goodness' or 'not-goodness', these components make one confused about the quality of the building. Such type of confusion leads to a 'hesitation'. This part of the whole evaluation procedure is no doubt 'indeterministic' for the time being or for the present evaluator. Due to this 'indeterministic' part it may not be possible to complete the evaluation of the problem by the present evaluator. As a result it is difficult to assign some membership value to some component of the object. To overcome such type of situation an extended and alternative fuzzy set theory named as IFS was initiated by Atanassov [1].

The IFS has enriched its potentiality to deal with vagueness in some special situations. Fuzzy sets are intuitionistic fuzzy sets but the converse is not necessarily true. There are many real life problems where intuitionistic fuzzy sets are more useful to deal with. Suppose that a human being wants to express his opinion about the beauty of a flower. According the concept of classical set theory, he has only two options to classify the beauty of the flower. He may choose either 0 or 1, i.e, the flower may be either ugly or beautiful. But, it may not be the real fact. he may not be fully dissatisfied or fully satisfied about the beauty of the flower. In fuzzy set he has the liberty to express his view that the flower is beautiful with a grade of membership 0.8, say, which determines the grade of non membership as 0.2. The real situation may not be so; a flower may have the beauty with grade of membership 0.8, but not ugly at all, i.e., its grade of nonmembership has the value 0. Therefore, one needs a most appropriate approach which would help to deal with such a problem whose solution depends upon human perceptions. Thus, IFS is a very useful mathematical tool used in many situations having imprecise, vague, inexact data.

An intuitionistic fuzzy set or an IFS A of X is an object having the form
$A = \{ < x, \mu_A(x), \nu_A(x) > | x \in X \}$,
where the function $\mu_A : X \rightarrow [\,0, 1\,]$ and $\nu_A : X \rightarrow [\,0, 1\,]$, define the degree of membership and degree of non-membership respectively of the element $x \in X$ to the set A, and for every $x \in X$

$0 \leq \mu_A(x) + \nu_A(x) \leq 1$.
Here, for x $\in$ X, the value $\mu_A(x)$ signifies the membership value of x in A evaluated some how; where $\nu_A(x)$ signifies the nonmembership value of x evaluated by identical evaluator and the amount $\pi_A = 1 - \mu_A(x) - \nu_A(x)$ is termed as intuitionistic index or hesitation index and is the degree of indeterminacy concerning the membership of x in A.

1.1.4 Rough Sets

Pawlak [46] introduced a new tool called 'Rough Set Theory' to deal with uncertainty. The main advantage of rough set theory is that it does not need any preliminary or additional information about data (like prior probability in probability theory, grade of membership or value of possibility in fuzzy set theory). Other advantages of the rough set approach include its elegancy in handling and its simple algorithms.

The theory of rough sets [46] is regarded as a major mathematical tool for dealing with uncertainty that arises from granularity in the domain of discourse. In other words it deals with uncertainties which arise from the indiscernibility between objects in a set. This theory is an useful extension of the classical set theory. The concept of rough sets differs essentially from the ordinary concept of the set in the sense that for the rough sets we are unable to define uniquely the membership relation. There is another important difference between these concepts, namely that of equality of sets. In set theory two sets are equal if they have exactly the same elements. In case of a rough set the equality is termed as approximate (rough) equality. Thus two sets can be unequal in set theory, but can be approximately equal from the view point of rough sets. This is an important feature for a practical point of view, for often by using the available knowledge we might be unable to tell whether two sets are equal or not (i.e. they have exactly the same elements), but we can only say that, according to our state of knowledge, they have close features which are enough to be assumed approximately equal.

The rough set concept is founded on the assumption that with every object of the universe of discourse there is associated some information. For example, if objects are potential projects, their technical and economic characteristics form information (description) about the projects. Objects characterized by some information are indiscernible (similar) in view of available information about them. The indiscernibility relation generated in this way is the mathematical basis of rough sets theory. Any set of indiscernible objects is called elementary sets. Any subject of the universe can either be expressed precisely in terms of elementary sets or rough sets only. In the latter case, this subject can be characterized by two ordinary sets called lower and upper approximations. The lower approximation contains objects surely belonging to the subset considered, the upper approximation contains objects possibly belonging to the subset considered. An important advantage of the rough set approach is that it can deal with a set of inconsistent examples, i.e., objects indiscernible by condition attributes but discernible by decision attributes. Moreover, it provides useful information about the role of particular attributes or their subsets in the approximation decision classes, and prepares the ground for generation of decision rules involving relevant attributes.

The rough set theory is specially well suited to deal with inconsistencies in the process of machine learning. The fundamental notions of rough set theory is to compute lower and upper approximations. The rough set approach is based on knowledge with granular structure which is caused by the situation when objects of interests cannot be distinguished and they appear to be identical.

The mathematical basis of rough set theory is to generate the indiscernible relation. Knowledge is represented in physical form, i.e., if it is in the form of a data table (attribute-value systems) then this kind of data table will be called an information system. Let R be an indiscernibility relation which is also an equivalence relation defined over the universe U. For any no-empty subset X of U, the sets

$\underline{R}X = \{ x : [x]_R \subseteq X \}$ and

$\overline{R}X = \{ x : [x]_R \cap X \neq \emptyset \}$

are respectively called the lower and upper approximation of X, where the pair $A = (U, R)$ is called the approximation space and $A(X) = (\underline{R}X, \overline{R}X)$ is called the rough set of X in A (here $[x]_R$ denotes the equivalence class of R containing x).

The set $BN_R(X) = \overline{R}X - \underline{R}X$ will be called the rough boundary of X in A.

$POS_R(X) = \underline{R}X$ is called positive region of X

$NEG_R(X) = U - \overline{R}X$ is called negative region of X.

For the sake of convenience, we denote a rough set $A(X) = (\underline{R}X, \overline{R}X)$ by $(\underline{X}, \overline{X})$

Let $A(X) = (\underline{X}, \overline{X})$ and $A(Y) = (\underline{Y}, \overline{Y})$ be two rough sets in the approximation space $A = (U, R)$. Then

(i) $A(X) \cup A(Y) = (\underline{X} \cup \underline{Y}, \overline{X} \cup \overline{Y})$

(ii) $A(X) \cap A(Y) = (\underline{X} \cap \underline{Y}, \overline{X} \cap \overline{Y})$

(iii) The rough complement of A(X) in (U, R) denoted by $-A(X)$ and is defined by

$$-A(X) = (U - \underline{X}, U - \overline{X})$$

(iv) $A(X) - A(Y) = (\underline{X} - \overline{Y}, \overline{X} - \underline{Y})$.

Pawlak's rough sets provides a systematic approach for classification of objects through an indiscernibility relation. When a universe of objects is described by a family of attributes, the indiscernibility of the objects can be based on the attribute values of these objects. When two objects have the same value over a certain group of attributes, we say they are indiscernible with respect to this group of attributes or have the same description consist of an equivalent class and all equivalent classes form a partition of the universe.

Partition or equivalent relation, as the indiscernibility relation in Pawlak's original rough set theory, is still restrictive for many applications [75]. Different partitions of a universe generate different lower and upper approximations. Furthermore, in rough set theory, the lower and upper approximations are dual to each other and therefore determine each other.
In recent year a lot of works relevant to our discussion have been carried out using the rough sets [20, 21, 23, 26, 30, 36, 38, 39] and [28].

The new theory, the theory of soft set [37] by Molodtsov, is a mathematical tool which is nothing but one type of approximations, do not need any type of lower or upper approximation. Hence it is free from the above difficulties.

1.1.5 Soft Set Theory

The universe consists of several kind of objects. The objects may be either living or nonliving. Any object we talk about is a fuzzy concept. When we say a cow, it could be a large cow, or a small one, a healthy cow or a thin, a young cow or an old cow. Because of our fertile imagination we are bombered simultaneously with many ideas and properties of the same object. Even when we speak of the same cow, we have it in mind yesterday or today, in the barn or out grazing. If we were to specify all the conditions we have in mind when we mean, "that" cow, we could run out of time and serve no good purpose. Even then we can never speak of the same thing at all time since time changes and the object may occupy different positions in space. In sum we always need a way to get at this elasticity in meaning, tightening and increasing for specificity to uniformize it across experience and operationalize it for practical usage.

The problem is compounded with the use of a highly limited language in which the same word is used for many purposes whose meaning, no matter how standardize evokes different feelings and hence different meaning and interpretation from individual to individual and sometimes in the same individual.

The boundaries of properties and even objects appear fuzzy because they are a part of a continuum whose boundaries merge into one another. We cannot just discard the idea of boundaries, because we need to decompose systems into parts, so we can understand what flows in them and out of them at what we call boundaries. It is this analytic process of discretization and decomposition that creates problems and difficulties for us. If we wish to cope with this type of fuzziness we must treat reality as a whole in systems terminology and minimize decomposing it into parts.

The problem of boundaries has to do with the degree of belongingness to the sets. For each element of a set the question is not whether it has or does not have a given property (i.e. a rose being red or a pink) but how strongly it has the property. A fuzzy property such for example as beauty is an aggregation a more specifically a hierarchy of various properties of different importance. To draw a conclusion about the beauty of a flower we may not explicitly mention the degree of beautifulness of it due to the lack of a suitable mathematical tool. We do not believe that there exists any single method which is able to model all types of uncertainty equally well. Most of the established theories and methods for uncertainty modelling are focused either on specific 'types of uncertainty' defined by their causes or they at least imply certain causes and they also require specific types or qualities of information depending on the type of information processing they use. One could consider these uncertainty methods and their paradigms as glasses through which we consider uncertain situations or with other words, there is no 'probabilistic uncertainty' as distinct from uncertainty. Uncertainty is considered as a situational property of phenomena which has various causes and which is also influenced by available number of theories, methods or paradigms to model uncertainty. To name just a few: probability theory, fuzzy set theory, rough set theory etc. In his pioneer work, soft set theory, Molodtsov [37] showed that all the above theories have their own difficulties. To avoid such inherent difficulties one must use an adequate parametrization.

Let U be an initial universe and E be a set of parameters. A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In the approach of soft set the initial description of any object has an approximate nature and we do not need to introduce the notion of exact solution. In several fields of sciences, engineering, economics etc. this newly proposed soft set theory will be very convenient and easy to apply in many problems in practice due to the absence of any restrictions on the approximate descriptions. In this concept the parametrization is done with the help of words, sentences, real numbers, functions, mappings and so on. Although this theory is at a beginning stage its application in various fields viz., operations research, analysis, differential equations etc., as already shown by Molodtsov. He gave an indication of the usefulness of the theory in decision making and other problems.

In game theory the basic problem is to maximize the pay function. But to construct a pay function is not a easy job. It is rather easy to describe a human behaviour directly showing the set of strategies which a person may choose in a particular situation. A choice function which associates the set of strategies with a given situation is developed in this theory. But this choice function is unable to give approximate description. Molodtsov described the person's behaviour with the help of s-function. For any set of ϵ-optimal choices and some approaches have been proposed for different types of uncertainties.

Since soft set is a collection of approximate descriptions of an object, the exact description of the object in not necessary. Molodtsov developed the definitions of upper softlimit, lower softlimit and softlimit of a function f. Molodtsov [37] also initiated the notion of the 'soft approximator' (which is a analogue of classical differential), the concept of the upper approximator, lower approximator and discussed its properties. For developing soft analysis, one should not start with the classical concept of the limit. The soft approximators are more convenient to deal with uncertain information and approximate calculation methods than the classical differentials due to its smoothness and regularization. The soft analogue of integral has also been given with sufficient conditions for existence. The stability of soft integral is also proved.

1.1.6 About Our Work in This Thesis

The digital revolution has made digitized information easy to capture, process, store, distribute and transmit. With significant progress in computing and related technologies and their ever-expanding usage in different walks of life, huge amount of data of diverse characteristics continue to be collected and stored in database. The rate at which such data are stored is growing phenomenally. Today, data are no longer restricted to tuples of numeric or character representations only. The advanced database management technology of today is enable to integrate different types of data, such as image, video, text, and other numeric as well as non-numeric data, in a provably single database in order to facilitate multimedia processing. As a result, traditional adhoc mixtures of statistical techniques and data management tools are no longer adequate for analyzing this vast collection of mixed data.

Data mining is a form of knowledge discovery essential for solving problems in domains involving large volumes of data. The individual data sets may be gathered and studied collectively for the purpose other than those for which they were originally created. New knowledge may also be obtained in the process, while eliminating the cost of additional data collection. Besides, data often exist in vast quantities over the internet in an unstructured format. The application of data mining facilitates systematic analysis in such cases and helps the user in extracting relevant information. Sometimes different kinds of data can be interspersed for better semantic representation, and often data may be erroneous.

Soft computing is a consortium of methodologies that works synergistically and provides, in one form or another, flexible information processing capability for handling real-life ambiguous situations. Its aim is to exploit the tolerance for imprecision, uncertainty, approximate reasoning, and partial truth in order to achieve tractability, robustness, and low-cost solutions. The guiding principle is to devise methods of computation that lead to an acceptable solution at low cost, by seeking for an approximate solution to an imprecisely or precisely formulated problem. Recently various soft computing methodologies have been applied to handle the different challenges posed by data mining. The main constituents of soft computing, at this juncture, include fuzzy logic, neural networks, genetic algorithms, rough sets, and signal processing tools such as wavelets. Each of them contribute a distinct methodology for addressing problems in its domain. This is done in a cooperative, rather than a competitive, manner. The result is a more intelligent and robust system providing a human-interpretable, low-cost, approximate solution, as compared to traditional techniques.

Pawlak's rough set theory [46] is one of such strong mathematical tool to deal with reasoning about uncertain or vague knowledge. Rough sets have often been compared with fuzzy sets [66], sometimes with a view to introduce them as competing models of imperfect knowledge. Such a comparison is misfounded [48]. Indiscernibility and vagueness are distinct facts of imperfect knowledge. Indiscernibility refers to the granularity of knowledge, that effects the definition of universe of discourse. Vagueness is due to the fact that categories of natural language are often gradual notions, and refer to sets with smooth boundaries. Fuzzy set theory relies on ordering relations that express intensity of membership. Rough set theory is based on equivalence relations describing partitions made of classes of discernible objects. So one theory is quite distinct from other, and they display a natural complementary as established by Dubois and Prade [20]in 1989.

In 1985, Pawlak [47] introduced a method of finding dependency of attribute based on indiscernibility relation. In 1983 Novototny and Pawlak [38], Orlowska [40, 41, 43], in 1985 Pawlak and Rauszer [50], in 1988 Grymala-Busse [26] described it in more details. Many researchers generalized the notion of rough sets. This generalization has been done by using the non-equivalence relations [42, 51, 56, 57, 59, 61, 63, 64, 65]. It is possible to obtain the upper and lower bounds by eliminating the transitivity, reflexivity and symmetry axioms. In [62] Yao studied the general properties of rough sets resulted from these axioms. The notion of approximation operators can also be generalized by using a covering of the universe [42, 51]. Lin [32, 33] proposed a more general frame work for the study of approximation operators by using the so called neighbourhood systems from a topological space and its generalization is called Frechet (V) space. In a neighbourhood system, each element of a universe is associated with a family of subsets of the universe. This family is called neighbourhood system of the element and each member in the family is called a neighbourhood of the element. Any subset of the universe can be approximated based on neighbourhood systems of all elements in the universe. With respect to a binary relation, the successor elements of a given element may be interpreted as its neighbourhood [61, 62, 63, 64, 65]. The theory of rough sets built on binary relations may therefore be related to neighbourhood system. A significant research on this topic has been done by Yao [61]. Properties of neighbourhood operators systems and rough set approximation operators are investigated in [60]. In chapter 2 we applied the soft set concept to solve a decision making problem by rough set approach which finds similarities to the approaches of Yao [61], Thielle [58] and Lin [29].

The set of parameters play a key role in soft set theory. These parameters may consists of words or sentences, real numbers, functions, mappings etc. So, fuzziness exists in the set of parameters to some extent. Considering this point of view we are motivated to define fuzzy soft sets. Fuzzy soft sets with some algebraic operations have been discussed in chapter 3 in detail. In our daily life moments we faced with some vague, inexact, imprecise informations. Zadeh's classical concept of fuzzy sets (or fuzzy Subsets) [66] is considered as a strong mathematical tool to deal with such type of vagueness. Since the initiation of fuzzy sets theory, there are suggestions for non-classical and higher order fuzzy sets for different specialized purposes. Some of them are 'two-fold fuzzy sets' proposed by Dubois and Prade [21], 'L-fuzzy sets' by Gougen [24].

Since there is no universal formula to determine the membership values of fuzzy sets or even for a particular situation, the membership values cannot always be determined as a result of insufficient available information. Similarly, while determining the non-membership values same problem arises, a part of such estimation naturally remains indeterministic. The indeterministic part has not been considered in case of fuzzy set theory but it has an important role in case of intuitionistic fuzzy sets. Considering the parameters of soft sets as intuitionistic fuzzy we are motivated to define intuitionistic fuzzy soft sets in chapter 4. We also develop some algebraic operations in intuitionistic fuzzy soft sets. We study the intuitionistic fuzzy relations [9, 10]. We also study the Sanche's approach [53, 54, 55] for medical diagnosis and the concept is generalized by the application of intuitionistic fuzzy sets theory. Using the technique 'similarity measurement method' we solve a decision making problem in intuitionistic fuzzy soft sets.

In chapter 5 we define some new operations viz., 'Dot', '+' etc. on intuitionistic fuzzy soft set and prove some propositions based on these newly defined operations. A decision making problem has been solved in this chapter.

Lastly, in the concluding chapter 6 we have made some general conclusions and also proposed some future plans of our research.