CHAPTER 1

INTRODUCTION

1.1. GENERAL INTRODUCTION

Queueing theory is a powerful tool in the analysis of waiting line models arising in real life situations such as railway reservation counters, banks, hospitals, and telephone traffic etc. Its application is not only restricted to the traditional areas, but has also been extensively used in manufacturing, production-assembly and inspection operation, maintenance, construction and mining, and computer and communication systems etc. In recent years, its importance has further increased in the area of telecommunications due to the developments taking place in Broadband-Integrated Service Digital Network (B-ISDN) which is intended for high speed transfer of video, voice and data into a common format. For the analysis and design of these systems one needs to know about various performance measures such as mean queue length, mean waiting time, blocking probability, server utilization, and system capacity etc. These and other related informations can be obtained by developing an appropriate queueing model and then solving it analytically as well as numerically using efficient mathematical techniques. Though mathematical techniques are a prerequisite for developing a queueing model, the veracity of the algorithms can be best tested by subjecting it to computational analysis. The increased use of queueing models in modern technology warrants such a treatment. Lately, the research in queueing theory has turned more application oriented, taking computational aspects of queueing models in its folds.

1.2. REVIEW OF PREVIOUS RESEARCH ON COMPUTATIONAL ASPECTS

In the past several researchers have carried out analysis of queueing models and derived analytical results using various mathematical techniques. However, analytical results of most of the queueing models have been presented in terms of probability generating functions (pgfs) or Laplace-Stieltjes transforms (LSTs) or in other complex mathematical forms etc. Moreover, their expressions, except in some simple cases are not at all possible to use for evaluating distributions of numbers in systems. In fact, one may ask what good is the model if its implementation is difficult? Many a times practitioners of queueing theory have also criticized about the applicability of theoretical results. Realizing this fact some researchers have started bridging the gap between the existing analytical results and their applications by practitioners who neither have time nor expertise to understand the mathematical details of the model. Cohen (2nd ed. 1982, p. 640) states that a rather neglected area is that of the development of algorithmic and numerical methods for queueing models. Neuts (1989, p. IX) quotes "A good numerical example is worth ten thousand words." Since last 15 years there has been a considerable interest in developing various techniques for the analysis of queueing models so that meaningful and easily computable results could be obtained. Also the recent emphasis on computational probability is increasing the value of queueing models in several areas such as computer and communication systems, computer networks, and flexible manufacturing systems etc. It is becoming standard for modelling and analysis to include algorithms for computing probability distributions of interest. Hence, the numerical aspects of queueing models are as important as their analytical developments.

The recent research trend especially on computational aspects of queueing models has grown tremendously. Numerical computations in queues is to focus attention on the numerical aspects of queueing models so that the final results could be fruitfully used by practitioners, queueing theorists and many others. Some systematic work in this direction has been done by quite a few researchers. Bagchi and Templeton (1972) have analyzed the bulk queues using numerical convolutions. Powell (1981) carried out analysis of bulk arrival and bulk service queues related to transportation and presented variety of numerical results. The matrix-analytic method, developed by Neuts (1981, 1989), has become a very powerful technique to obtain numerical results of many queueing models. The state-reduction (SR) algorithm for computing the steady-state probabilities of a finite-state, ergodic Markov chain has been developed by Grassmann, Taksar and Heyman (1985). Tijms (1986) in his book studied in great detail about the numerical aspects of queueing models, see also Tijms (1988). Brière and Chaudhry (1987) developed root finding technique to give a close form analytic expressions in terms of roots of the so called characteristic equation (CE) of bulk arrival queues. Subsequently the technique has been used to generate the numerical solutions of number of queueing models, see e.g., Brière and Chaudhry (1989), Chaudhry, Gupta and Agarwal (1991), Chaudhry and Gupta (1992) and Zhao (1994). The numerical solution of the waiting time, idle time distributions and departure process of the GI/G/1 queue has been discussed by Jain and Grassmann (1988), Grassmann and Jain (1989), and Chaudhry, Agarwal and Templeton (1992). Recursive computation for state probabilities of finite Markov processes has been introduced by Yang, Posner and Templeton (1991). They have also discussed some applications related to queues. Abate and Whitt (1992 a and b) have used the Fourier-series method for numerically inverting Laplace transforms and probability generating functions in queueing problems.

Though the above literature survey on computational aspects of queueing models was restricted to the steady-state solutions, however, quite a few researchers have developed efficient methods to obtain transient solutions of Markovian queues. Sharma (1990) in his book studied the transient behavior of number of Markovian queueing models. Parthasarathy (1987), Parthasarathy and Sharafali (1989), and Mohanty, Haghighi and Trueblood (1993) have also contributed.

1.3. MOTIVATION AND OBJECTIVE OF THE THESIS

The above literature survey clearly indicates that there is a further scope to carry out research work especially on computational aspects of queueing models. As there are large number of queueing models both with finite and infinite waiting space for which it is difficult to evaluate distributions of numbers in system as well as performance measures, if not impossible. This motivated us to do further research in this area. In fact, in this thesis we have taken an opportunity to carry out analytical and numerical studies of various queueing models with finite waiting space and/or finite source and we feel that we have achieved quite a good deal.

Some of the techniques mentioned in Section 1.2, have their own advantages and disadvantages. Moreover, there are other techniques which can be exploited to obtain analytical and numerical results of queueing models in a more efficient way. In the queueing literature supplementary variable technique is a well known technique which was initially used by Kosten in 1942 (see Kosten (1973)), but the wide use of this technique came about due to Cox (1955). In the past it has been extensively used to obtain analytical results of number of queueing models by considering elapsed service/interarrival time as a supplementary variable. But very few researchers have considered supplementary variable as a remaining service/interarrival time. Henderson (1972) used it for the steady-state analysis of $M/G/1/\infty$ and $GI/M/1/\infty$ queues, Hokstad (1975 a and b) used it to obtain steady-state and transient solutions of $GI/M/c/N$ and $M/G/1/\infty$ queues respectively. Again the focus of all these studies except few were mainly restricted towards the theoretical developments of the models from which numerical results are difficult to obtain. However, Baba