CHAPTER - I

DETERMINATION OF FIELD DISTRIBUTION AND EVALUATION OF EDDY-CURRENT LOSS IN SOLID IRON SUBJECTED TO TWO FREQUENCY EXCITATION - A NUMERICAL METHOD OF ANALYSIS

1.1 INTRODUCTION

The analysis of field distribution and the evaluation of eddycurrent loss in solid and laminated iron subjected to alternating excitation has been a topic of continuing interest. (References 2,4,23,25-31,34,35,38,39,43,45,51). The main factor that complicates the analysis is the nonlinear nature of the B-H characteristic of the material. Of the different methods of analysis developed by various authors to take this nonlinearity into account, two are significant. These are (i) the graphical method due to Pohl[39] and (ii) the limiting nonlinear analysis developed by Maclean[30], McConnell[31] and Agarwal[2]. However, results of the above analyses are restricted to the case when the material is subjected to an alternating magnetic field at a single frequency.

A problem of further interest is the evaluation of eddy-current loss when the material is subjected to two alternating magnetic fields of different frequencies. This is an extremely complex problem and no attempt has so far been made to investigate the behaviour of the core material when it is subjected to two frequency excitation. The nature of field distribution inside the core under two frequency excitation conditions differs considerably from that under single frequency excitation conditions. This aspect is considered in some detail in this chapter.

In order to understand the nature of the problem, consider an infinite half space of iron with its surface on the y-z plane and excited such that the magnetising force at the surface is in the y direction only. At any point inside the material H_y, B_y and J_z alone exist and each is a function of x and time t only. The Maxwell's equations governing the field quantities are

$$\frac{\partial H_y}{\partial x} = J_z$$

$$\frac{\partial J_z}{\partial x} = \frac{1}{\rho}\frac{\partial B_y}{\partial t} \qquad \ldots \; 1.1$$

leading to

$$\frac{\partial^2 H_y}{\partial x^2} = \frac{1}{\rho}\frac{\partial B_y}{\partial t} \qquad \ldots \; 1.2$$

where ρ is the resistivity of the material. Since, H_y, B_y and J_z alone exist, the suffixes can be dropped for convenience. The functional relationship between B and H is governed by the magnetisation characteristic of the material and can be written as

$$B = f(H) \qquad \ldots \; 1.3$$

Eqn. 1.2 then becomes

$$\frac{\partial^2 H}{\partial x^2} = \frac{1}{\rho}\frac{\partial B}{\partial H}\frac{\partial H}{\partial t}. \qquad \ldots \; 1.4$$

Consider the single excitation case for which the boundary condition at the surface is given by

$$H^s = \hat{H}^s \sin\omega_a t \qquad \ldots \; 1.5(a)$$

If the B-H relationship is linear, the solution of eqn. 1.4 for the surface magnetising force given by eqn. 1.5(a) is straight forward. On the otherhand, if the B-H relationship is nonlinear, no rigorous analytical solution exists for an arbitrary B-H relationship and one has to resort to graphical[27] and numerical methods[18,29,51]. These solutions reveal the fact that, when the B-H relationship is nonlinear, the waveform of H, besides attenuation, getsdistorted such that the field quantities inside the material will consist not only of the fundamental but odd harmonics of the forcing frequency also.

Consider next, the two excitation case for which the boundary condition at the surface is given by

$$H^s = \hat{H}_a^s \sin\omega_a t + \hat{H}_b^s \sin(\omega_b t + \gamma) \qquad \ldots \quad 1.5(b)$$

where ω_a and ω_b are the low and high frequencies respectively. Exact nature of the field distribution for this case has not been examined in any detail so far. However, based on the behaviour under single excitation condition, one can anticipate the following nature for the field distribution inside the material.

(1) In general, the field quantities inside the material will be of frequencies $(\mp p\omega_a + q\omega_b)$ where p and q are integers including zero.

(2) The B-H relationship being an odd function of H, (p+q) will always be odd.

(3) The frequency components can further be divided into three categories.

(a) Fundamental and harmonic components of ω_a: $q = 0$ and p assuming odd values including unity correspond to this case.

(b) Fundamental and harmonic components of ω_b: $p = 0$ and q assuming odd values including unity correspond to this case.

(c) Combination frequencies: Frequencies not covered by (a) and (b) correspond to combination frequency components. When the frequency ratio is an integer the combination frequency components will coalesce with the harmonics.

Determination of the several frequency components of field quantities mentioned above, would give a clear picture of field distribution inside the material. However, from a practical point of view, the object of main interest is, often, the evaluation of the total eddycurrent loss in the material. This can be directly obtained by determining the power transferred into the material corresponding to the surface excitations at ω_a and ω_b. If $\hat{E}_a$ and $\hat{E}_b$ are the amplitudes of the fundamental components of E at the surface at frequencies ω_a and ω_b, the power transferred at frequencies ω_a and ω_b, namely, P_a and P_b are given by

$$P_a = \tfrac{1}{2}\, \hat{E}_a\, \hat{H}_a^s \cos \phi_a \;,\; P_b = \tfrac{1}{2}\, \hat{E}_b\, \hat{H}_b^s \cos \phi_b$$

where $\cos \phi_a$ and $\cos \phi_b$ are the power factors corresponding to $\hat{H}_a^s$ and $\hat{H}_b^s$.

One should expect the amplitude and phase of the field quantities of the different frequency components, as well as the total eddy current loss inside the material to depend upon

(i) the amplitudes of both H_a and H_b ,

(ii) the frequencies ω_a and ω_b , and

(iii) the phase angle γ between them.

The main object of the present investigation is to bring out, in quantitative terms, the effect of the above three factors on (a) the distribution of the different frequency components and (b) the eddycurrent loss and power factor due to each of the exciting forces at the surface. Since rigorous analytical solutions do not exist for this problem, a numerical method of solution to the problem on hand is developed in this chapter.

An implicit numerical method originally due to Crank-Nicolson[14] has been effectively used to study the field distribution under single frequency excitation case[1,18]. The method leads to a tridiagonal system of equations which can be systematically solved by Gauss elimination and back substitution. In this chapter, this method is reviewed and extended to two frequency excitation case.

50 Hz is chosen as the base (low) frequency and solutions are obtained for certain specific frequency ratios, both integral and nonintegral, ranging from 1.8 to 19 - assuming a Frohlich characteristic for the B-H curve. A wide range of $\hat{H}_b^s / \hat{H}_a^s$ has been chosen for solution, for a maximum value of $\hat{H}_a^s$ (or $\hat{H}_b^s$) equal to 16000 AT/m, a value well within the saturation region of the B-H curve.

Based on these results certain significant conclusions have been drawn regarding the behaviour of the material under