Abstract

As is well known, the mathematical theory of reliability had developed and grown as a result of operational experience with complex military equipment and modern technological systems. In 1980's, a great deal of reliability research was concerned with the development of network reliability evaluation techniques. The early Advance Research Project Agency (ARPA) network, the forerunner of today's Internet and Word Wide Web, further provided fillip this effort.

A standard model of a communication system or a network is a graph in which nodes represent communications centers and edges represent links between communication centers. Along the same concepts, a technological system can also be modeled, which in effect turns out to be a graph or a *network* with edges and nodes. This model in large technological system may in reality turn out to be much more complex than just a simple series-parallel configuration.

One of the measures of assessing the performance of such a system is by computing the *terminal reliability* (TR), which ensures connectivity between two specified *points* or *nodes* in a system. This is an indication of failure free operation of all elements between two specified nodes representing *input* (or *source*) and *output* (or *sink*) of the system. Similarly, in communication systems, it is often desired to establish communication between two nodes or *terminals* of the network where the links and nodes can have two states (good or bad) and *two-terminal reliability* (TR) of such a system assumes importance and is an essential parameter in assessing its performance. Basically, the problem of computing TR is a NP-hard problem and has for long drawn the attention of

researchers all over the globe to explore the possibility of developing efficient algorithms for computing TR.

There are other measures, such as *all-terminal* and *k-terminal* reliabilities, of assessing the network reliability as well. *all-terminal reliability* is defined as the probability that every node of the network must be able to communicate with all other nodes of the network, whereas *k-terminal* reliability is the probability that a specified set of *k-nodes* of the network are able to communicate (or exchange information, data or energy) with each other. In fact, these measures are the generalized form of two-terminal reliability measure.

In this thesis, the author has considered the problem of obtaining compact forms of two-*terminal, all terminal* and *k-terminal* reliabilities using multi variable inversion (MVI) approach besides addressing himself to some significant aspects of *multi state systems* (MSS). The author has also tried to ensure that a user is able to solve a large system reliability evaluation problem on a low-end computer or a personal computer. Since it is difficult to specify the *largeness* of a system that can be specified in advance, it is considered enough to demonstrate if the author is able to solve a relatively large problem on a personal computer than is possible by other methods.

Another attempt the author has made in this thesis is to be able to provide an integrated approach to the problem of reliability evaluation using a common specific framework. There are several phases in the process of system reliability evaluation, such as:

- Reducing the large system to a non-reducible model that can be handled easily
- Determining path sets or cut sets of this non-reducible form and
- To determine the most compact form of system reliability expression incorporating MVI approach in order to reduce round off errors during system reliability computation.

In this thesis, all these phases are accomplished using a common platform of connection matrix representation of the system as will be seen, as we browse through the thesis. It can be easily realized that connection matrix is the most compact form of representation of a system model than is possible by any other means. Thus the thesis provides a unifying or an integrated approach to the reliability evaluation problem of large and complex system and be able to attempt to handle a large problem on a personal computer.

One of the earlier methods of computing two-terminal reliability of series-parallel (SP) networks was proposed by (Misra and Rao 1970) and is known as Misra Matrix Method (MMM) in the literature. The method is still considered to be the fastest and the most efficient approach to date for computing reliability of SP systems. However, the method, in its original form, was supposed to be applicable to the systems comprising of parallel redundancies with exponential failure distribution of the constituent elements. In this thesis, the applicability of Misra Matrix method is extended not only to more general types of SP-networks consisting of redundancies such as *k-out-of-m* and *standby* besides *parallel active* redundancies but also the elements of the system may have any arbitrary failure distributions such as exponential, Rayleigh, Weibull or/and extreme-value.

The most widely employed approach for evaluating the reliability of complex networks use path sets or cut sets of the networks. However, the main draw back of path set (or cut set) approach happens to be the steep rise in the number of path sets in a highly connected or complex network. In fact the number of path sets in a well-interconnected network can be quite large and unionizing these path sets to obtain a system reliability expression can be cumbersome and sometimes unwieldy.

In this thesis, the author proposes efficient methods for enumerating minimal path sets and cut sets using connection matrix of the network without performing any matrix inversion or multiplication operations as demanded by some of the earlier methods. The path sets method presented in this thesis generates minimal path sets in increasing order of cardinality in lexicographic order, either in terms of either network nodes or links. This process of generation affects tremendous reduction in number of disjoint terms, subsequently, while determining the system reliability expression. Similarly, cut sets enumeration algorithm proposed in this thesis generates minimal cut sets by operating on the connection matrix following certain rules and does not require generation of minimal path sets or basic path sets as a prerequisite for generation of cut sets as was common with earlier methods [Locks 1978, Rai 1979, Shier and Whited 1985).

The most popular and useful path (cut) sets methods in the literature are those, which obtain the reliability expression in terms of the sum of disjoint products. The techniques based on *sum of disjoint products* (SDP) have been used to produce a compact TR expression with minimum number of terms. Here again, several techniques have been suggested in the literature in the past but most efficient and effective are considered to be those, which use multiple variable inversion (MVI) to obtain disjoint terms. In MVI, each of the term of expression covers a greater domain of Boolean structure function. This feature in effect reduces the round-off errors generated on account of multiplications involved in computation of numerical value of system reliability. However, MVI approaches are still in the developing stage and research is still progressing in the direction of enhancing the speed of algorithms and minimizing the number of disjoint terms generated to obtain a compact TR expression.

Combining the best features of the existing best techniques, namely, CAREL and KDH88, the author presents two new MVI techniques for obtaining the most compact form of system reliability expression in this thesis. The efficacy of the proposed algorithms is amply demonstrated by computing TR expression and numerical values for 13 test networks of varied complexities. The proposed methods produce results very fast even on a low-end desktop PC, which was not possible with the existing techniques.

The MVI methods have also been applied for evaluating *k-terminal* reliability and *all-terminal* reliability of a network. In this regard, too, both the methods, viz., HM-1 and HM-2, outperform the existing MVI methods.

Very often, in reliability analysis, particularly of large maintained systems, the operation of the system may have to be represented by a state diagram or transition matrix, which involves the states and rate of departure from or arrival to these states in the system dynamics. Transitions may be determined by a variety of possible events, e.g., the failure or repair of an individual component. A state-to-state transition is characterized by a probability distribution. Under reasonable assumptions, the system operation may be analyzed using a Markov model.

In this thesis, a generalized procedure of developing state transition diagram or transition matrix of a multi-state and multiple redundant units (similar or dissimilar) has been developed with the help of Markov model. Some of the issues not addressed earlier in the literature, have been discussed in this thesis. This would facilitate the analysis of a large complex maintained system. A notion of plates representing a system geometry has also been proposed.

In summary, the author has made an humble effort in the direction of computing network reliability of large and complex systems and has provided some useful methodology for handling such systems. All algorithms proposed in this thesis have been tested on several test networks of varied complexities and appropriate programs have been developed on *Matlab* platform.