Abstract

This thesis presents B-spline transforms for signal and image processing applications. A B-spline interpolation algorithm has been introduced using simple convolution operations with the complexity $O(N)$. A *quasi-interpolation* scheme has been derived for on-line interpolation applications. Certain linear spaces of generalised polynomial splines (called generalised B-splines) that can have several applications including interpolation, smoothing and the construction of optimal quadrature formulae have been presented. This concept of generalised B-spline (g-B-spline) functions is originated from the idea of tensioned splines. This thesis presents algorithms for the continuous representation of discrete signals in terms of g-B-splines (direct generalised B-spline transforms) and for interpolative signal reconstruction (indirect generalised B-spline transforms) with an expansion factor of m. The key to our approach is to use a family of g-B-spline filters. The scale-changeable g-B-spline bases have been used to derive such filters. It has been shown that the scale factor can be modified to tune the approximated filter bandwidths to achieve better approximation. Frequency response characteristics of these filters have been investigated. Experimentally it is seen that a perfect reconstruction of a band-limited signal can be obtained by using cubic g-B-spline bases. Application to image representation and interpolation has been considered. A general approach for performing the first-order derivatives of the images using regularised g-B-spline filters has been presented.

The work has been extended to make use of these results to describe an equivalent filtering interpretation (low-pass filtering followed by a g-B-spline interpolator) of g-B-splines together with least squares generalised B-spline signal approximation methods. This work also concern with new theoretical results showing factorisation of the transfer functions of the higher degree g-B-spline filters. Experimental results on image data compression and image pyramid generation are presented to reveal the suitability of the proposed scheme.

A suitable scheme for wavelet decomposition using recursive generalised IIR filters has been presented. Experimental results show that the use of generalised

IIR filters for image compression yields improved fidelity and SNR. An unsupervised texture segmentation algorithm based on the multi-channel filtering theory has been presented. The filter channels are characterised by a bank of Gabor like tuned modulated g-B-spline filters. A wide variety of images (both natural and artificial texture images) have been considered to validate the proposed scheme.

This thesis also deals with the concept of orthogonal transformation and proposes an orthogonal discrete spline transform (DSPLT) for processing and representation of signals in terms of modified spline basis functions. The principle of periodic B-spline interpolation is explored. Based on the *orthonormal* properties of the eigenvectors of the periodic B-spline interpolation matrix, a complete set of *orthonormal* modified spline basis functions which constitute the basis set for the DSPLT has been developed. An efficient VLSI algorithm for computing the discrete spline transform coefficients of variable length has been proposed. The spectral properties of the modified spline basis functions are explored. Dealing with a continuous signal representation, the empirical formulae for computing the spline transform coefficients of the integral or derivative of a function, whose discrete spline transform coefficients are known, have been derived. A novel interpolation scheme using the DSPLT technique has been presented. Experimental results on signal/image interpolation and representation are produced to highlight the significance of the proposed scheme. A multiresolution analysis (MRA) technique using *wave packets* in terms of the modified spline basis has been introduced.

Keywords: B-spline, B-spline interpolation, signal processing, image processing, IIR filters, recursive filtering, image interpolation, edge detection, image compression, image pyramids, wavelet transforms, Gabor filters, multi-channel filtering, texture segmentation, spline harmonic basis, *orthonormal* functions, discrete spline transform, algorithm theory, wavelet packets, multiresolution analysis.