# CHAPTER I

## 1.1 Introduction.

There is a progressively increasing tendency towards the introduction of the ultimate strength methods in reinforced concrete design. It has been in use for a fairly long time in Brazil and the U.S.S.R. and more recently it has been incorporated in the Codes in U.K. (1957), U.S.A. (1956-63) and India (1964) as an alternative to the elastic theory.

The concept, however, is not new : in reinforced concrete design which started at the end of the nineteenth century ultimate strength methods based on non-linear compressive stress strain diagrams were proposed along with the straight line theory. In view of the limited experience and laboratory tests available at that time the straight line method won acceptance because of its simple mathematics and adequate safety. While this approach has been and is still being used for beams, it was realised as early as 1920 that the design of columns has necessarily to be carried out on a more realistic basis as the observed stresses in columns bore no relation to those calculated by the straight line theory. This led to the classical investigation on axially loaded columns by Lyse, Slater and Richart followed by rational expressions for the strength of centrally loaded columns.* These inelastic design formulae proposed for columns loaded axially or with small eccentricities were readily incorporated in the various Codes although beams and columns with large eccentricities continued to be designed after the standard theory.

From the survey of a large volume of test data Whitney[1], in 1940 proposed the equivalent rectangular stress distribution in concrete for ultimate strength analysis of beams and columns. Hognestad's[2] extensive investigation on eccentrically loaded square and circular columns confirmed the general applicability of the ultimate strength theory to combined bending and axial load.

Interest in the ultimate strength design of columns subjected to axial thrust and bending in two planes followed soon after. The variables involved in such columns are large and it has not been possible so far to formulate general mathematical expressions to determine the ultimate strength as was done by Whitney and Hognestad for the simpler case of eccentrically** loaded columns. When the position of the

---

* The contributions of Lyse, Slater and Richart are well known and are chronologically listed in the bibliography of Ref.2 of this thesis.

** Columns having eccentricity with respect to only one principal axis will be referred to hereafter as 'eccentrically loaded columns'.

neutral axis is known or assumed, the magnitude of the ultimate load and its moments about the principal axes can be determined from the stress strain relations for steel and concrete with the aid of the equations of equilibrium. On the other hand, when the position of the neutral axis is not known to start with, the equations can be solved only by the method of successive approximations.

A brief review of the available methods of analysis and results of the tests are presented in the following sections.

## 1.2 Historical Review.

The available methods of analysis can be broadly classified into two categories :

i) The trial and error method ;

ii) Determination of the ultimate load from failure surfaces in columns.

The basic principle in the first method is to locate, by trial and error procedure, that particular position of the neutral axis for which the internal forces developed are in equilibrium with the external load.

In the second method, ultimate loads are determined from failure surfaces which define the load and its moments at any point in the column. The different failure surfaces and their significance are explained in due course.

### 1.2a Methods based on trial and error procedure.

A systematic procedure based on the successive approximation method was first suggested by Whitney and Cohen,[3] Fig.1.1 (page 6 ). The steps involved are :

i) Assume a section completely.

ii) Assume arbitrarily the position and inclination of the neutral axis.

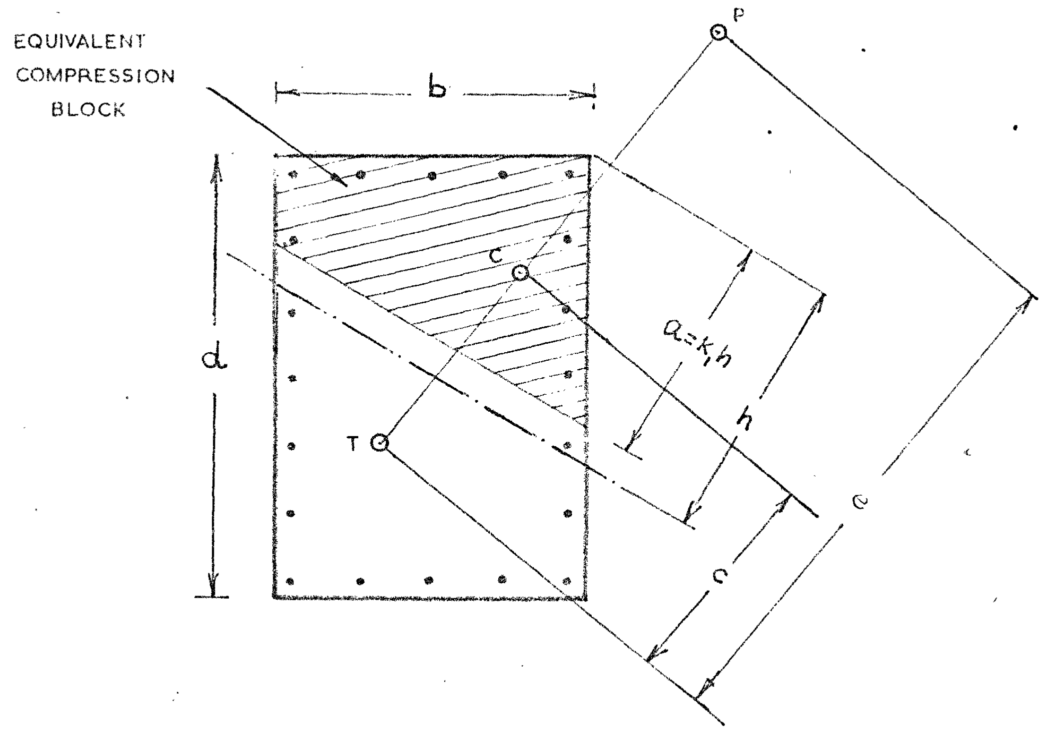
iii) Calculate the total compression in concrete and its centre of action assuming a uniform compressive stress of $0.85\ f'_c$ over the equivalent area of the compression zone.

iv) Compute the stresses in the bars assuming a maximum ultimate concrete strain in the outer most fibres in compression and determine the centre of action of forces in tensile steel and compression steel.

v) Check whether the internal forces and the external load satisfy the laws of statics.

vi) Repeat steps (ii) to (v) for a new position of the neutral axis, until the forces balance or it becomes obvious that the column section requires revision when the whole process is repeated for a new section.

vii) Continue the trial and error procedure till an acceptable section is obtained.



C = TOTAL COMPRESSION IN STEEL AND CONCRETE
T = TOTAL TENSION IN STEEL

FOR EQUILIBRIUM

$$P + T = C$$

AND $$Pe = C.c$$

FIG. 1.1 STRESS CONDITIONS AT FAILURE (WHITNEY)

Trial and error procedures are generally not convenient and they involve many cycles of computations, more so in biaxially loaded columns as the compression zone may vary from a triangle to a pentagon depending on the location of the neutral axis as in Fig. 1.2 (page 8 ). These remarks also apply to the other methods of the trial and error type.

Au[4] presented charts as aid to the design based on the recommendations of the ACI-ASCE Joint Committee on Ultimate Strength Design.[5] He formulated the equations of equilibrium for different positions of the neutral axis based on the following assumptions :

i) All the steel inside the equivalent compression area is stressed to yield point.

ii) The stresses in the bars close to the neutral axis may be neglected.

iii) All the bars in tension are acting under an average uniform stress $f_s$, i.e. the resultant tensile force is assumed to act at the centroid of the areas of the bars.

The axes of reference about which the internal and external moments are expressed are chosen to pass through the centre of gravity of the tensile bars.

According to this method an arbitrary position for the neutral axis is assumed and then the actual position is determined with the aid of parameters presented in charts. The trials