Abstract

The main objective of the thesis is to study various optimization problems such as mathematical programming problems, variational inequality problems, equilibrium problems under generalized convexity and generalized monotonicity assumptions in Riemannian manifolds.

The thesis is comprised of seven chapters along with a chapter containing concluding remark and scope of future research. Chapter 1 is the introductory chapter, where brief introduction to basics of manifold, geodesic, exponential map, (KKM) technique and adequate literature survey on optimization problems, generalized convexities, generalized monotonicities, variational inequality problems and equilibrium problems have been presented. In Chapter 2, we have introduced both (p, r)-invex and $\rho-(p, r)$-invex functions from functions defined on $\mathbb{R}^n$ to functions defined on Riemannian manifolds and have studied optimality conditions and duality results (weak, strong, converse duality) of a pair of optimization problems under these generalized invexity assumptions. In Chapter 3, we study the Karush-Kuhn-Tucker (KKT) type optimality conditions for an optimization problem on a Riemannian manifold where the objective function is geodesic η-invex, but the functions attached to inequality constraints are not necessarily geodesic η-invex. We also provide the concept of semistrictly geodesic η-prequasi invex functions on the said spaces and study their properties. Chapter 4 deals with the existence results of variational inequality and mixed variational inequality problems on Hadamard manifolds. Furthermore, we show that the sequence generated by the proximal point algorithm is well defined and converges to a solution. In Chapter 5, we investigate a projection-type algorithm for set-valued variational inequalities with pseudomonotone vector field on Hadamard manifolds. In Chapter 6, we solve the equilibrium and mixed equilibrium problems on Hadamard manifolds. In this chapter, we also introduce the implicit and explicit algorithms for mixed equilibrium problems. Chapter 7 deals with some existence theorems of different types of generalized vector equilibrium problems (GVEP) on Hadamard manifolds.

It has been observed that many nonconvex functions and nonmonotone vector fields can be transformed into convex functions and monotone vector fields, respectively, with the help of proper Riemannian metric. This fact draws the attention of researchers to extend techniques of nonlinear optimizations from Euclidean spaces to Riemannian manifolds. The theories that are developed in this thesis will draw the attention of researchers to work in this area.

Keywords: Manifold; Riemannian manifold; Hadamard manifold; Generalized invexity; Nonconvex optimization; Duality; Optimality condition; Variational inequality problem;

Equilibrium problem; Generalized vector equilibrium problem; Generalized monotonicity; Riemannian metric; Hemicontinuous vector field; Parallel transport; (KKM) mapping; Proximal point algorithm; Projection-type algorithm.