Synthesis of Asymmetric Multiport Network

Synopsis of the Thesis to be submitted in Partial Fulfillment of the Requirements for the Award of the Degree of

of

Doctor of Philosophy

by

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Contents

1 Introduction

Multi-port electrical network finds its usefulness in several practical applications like wireless communication, radar and power-delivery systems. In RF, microwave and millimeter-wave frequency range some of the commonly used multi-port networks are two-port filters and impedance matching networks (IMN), three-port power-dividers (PD), four-port rat-race (RRC) and branch-line couplers (BLC) etc. Impedance matching finds application in the design of antenna feed network, high frequency amplifier input and output matching, design of power combiners and dividers etc. The IMNs are finding increasing usefulness in the modern day applications like RFID and impedance compensation network for energy harvesting. RRC and BLC are used as power combiners and dividers, and find applications in the design of balanced amplifiers, image rejection mixers, feed networks for antenna arrays, modulators, demodulators, etc. In various cases such multiport networks may be composed of asymmetric elementary blocks for example to facilitate design of such networks in a given form-factor. Thus a systematic approach to synthesis of such multiport asymmetric networks is essential.

1.1 Literature Survey

Conventionally in high frequency regime such passive multiport networks are synthesized using several sections of transmission lines (TLs) [1]. In the existing literature the symmetric-TLs are replaced by equivalent symmetric two-port networks (TPN) for size reduction and harmonic suppression [2]-[3] or to achieve dual-band operation [4]. Recently efforts have been directed as in [5]-[6] to replace the symmetric-TLs by asymmetric-TPNs to obtain even better figure-of-merit than their symmetric counterparts. However in such a process, an equivalence with the symmetric TPN was made, which was fundamentally incorrect and pointed out in [7],[8]. While in the case of symmetric networks having a defined plane of symmetry, the derivation of design equations are performed using the efficient even-odd mode based decomposition; such an approach is not feasible for the asymmetric networks. Multiport networks having asymmetric-TPN [8] or unequal port impedance [9] does not have a defined plane of symmetry. To tackle this problem Ahn *et al.* proposed a technique to decompose the four-port networks like the RRC [9]- [10] and the BLC [10]- [11] into simpler three-port power dividers. Reduction of the four-port to the three-port was carried out by directly removing the isolation port and its associated connectivity, which is in general an incorrect procedure as has been shown in [12] and is a part of the discussion in this thesis.

Another issue of practical importance is the varaibility of port impedance. Tradional design of multiport networks are carried upon based on fixed port impedance i.e., 50 Ω. In various scenarios, the load or source impedance might vary and may even be complex. Therein lies the necessity of designing an IMN to connect such a source or load impedance

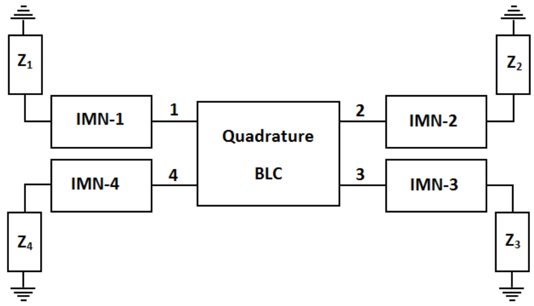


Figure 1: Quadrature BLC terminated by four different complex loads using different IMNs.

to a fixed port impedance, thus achieving maximum power transfer from the source or to the load. The principle of designing an IMN in the existing literature[13]-[14] are usually based on minimizing the reflection loss. Not much attention has been devoted to the transmission characteristics of the IMN, specially with regard to the transmission phase-shift. The importance of transmission phase-shift can be explained through the example given below.

Consider an example of a quadrature-BLC terminated by load with four different complex impedances Z_i ($i \in 1, 2, 3, 4$) through four different IMNs having transmission phase φ_i ($i \in 1, 2, 3, 4$). The S-parameters of this network can be written as

$$S_{new} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & e^{-j\left(\frac{\pi}{2}+\varphi_1+\varphi_2\right)} & e^{-j(\pi+\varphi_1+\varphi_3)} & 0 \\ e^{-j\left(\frac{\pi}{2}+\varphi_1+\varphi_2\right)} & 0 & 0 & e^{-j(\pi+\varphi_2+\varphi_4)} \\ e^{-j(\pi+\varphi_1+\varphi_3)} & 0 & 0 & e^{-j\left(\frac{\pi}{2}+\varphi_3+\varphi_4\right)} \\ 0 & e^{-j(\pi+\varphi_2+\varphi_4)} & e^{-j\left(\frac{\pi}{2}+\varphi_3+\varphi_4\right)} & 0 \end{bmatrix} \quad (1)$$

If $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4$, then the desired phase imbalance property between the ports of the network will remain unchanged even though the phase between the input and output will be shifted by $2\varphi_1$. If the transmission phase is not incorporated in the design of the IMNs and are therefore different, then the phase imbalance property between the ports of the network will change and the network will no longer have the quadrature property as in a BLC. Since the design equations of an IMN in the existing literature does not consider the transmission phase, it is difficult to predict the property of the new network. Henceforth incorporation of the transmission phase is crucial in the design of a matching network.

1.2 Objectives and Scope

1. Provide accurate synthesis equations of elementary asymmetric block based rat-race and branch-line coupler, thus correcting the fallacy in the existing literature.

2. In the existing literature transmission phase (between the input and output) has not been considered while designing couplers. A Synthesis approach has been provided

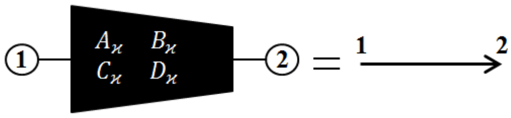


Figure 2: General (asymmetric or symmetric) two port network (TPN) used as building block and its directed representation in a network topology.

considering the true transmission phase and unequal power division at the output.

3. When a network does not have a plane of symmetry analysis or synthesis of the network is not possible using the convenient even-odd mode decomposition. This thesis will provide an alternate decomposition technique to synthesize multi-port network irrespective of plane of symmetry.

4. The transmission phase of an impedance matching network has not been considered in the existing literature. A general procedure of synthesis of such impedance matching networks incorporating the desired transmission phase into account has been provided.

2 Synthesis of RRC and BLC using Asymmetric-TPNs

2.1 Asymmetric Two-port Network (TPN)

As discussed above, the TPNs can be in general asymmetric in nature, and hence are best represented with an arrow shown in Fig. 2, where the arrow head and tail indicates port-2 and port-1 of the TPN respectively, whose ABCD parameters are as shown in Fig. 2. As the ideal RRC is lossless, passive and reciprocal, the ABCD parameters corresponding to the elementary TPNs are defined as,

$$\begin{bmatrix} A_\kappa & B_\kappa \\ C_\kappa & D_\kappa \end{bmatrix} = \begin{bmatrix} a_\kappa & jb_\kappa \\ jc_\kappa & d_\kappa \end{bmatrix} \tag{2}$$

with, $a_\kappa d_\kappa + b_\kappa c_\kappa = 1$. This type of lossless reciprocal TPN blocks with network parameter as in (2) can be implemented by any of the following two-port asymmetrical network shown in Fig. 3.

2.2 Rat-race Coupler (RRC)

The simplest method for synthesizing RRC using TPNs (Fig. 4(b)) from the conventional TL based RRC (Fig. 4(a)) is by equating the network parameters of the TPN block (symmetric or asymmetric in nature) to that of the (symmetrical) $\lambda/4$ TL. This is fundamentally wrong if the TPNs are asymmetric in nature and therefore leads to incorrect result [5]-[6]. However, one can equate the four-port Scattering parameters of the

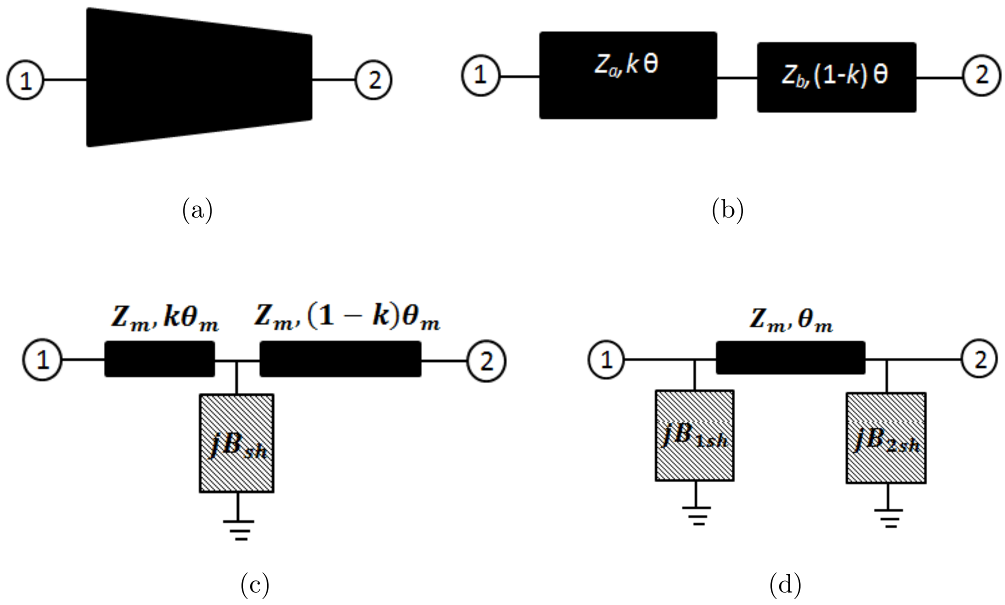


Figure 3: Various asymmetric TPNs: (a) tapered line, (b) stepped-impedance line, (c) T-Network, (d) Π-Network

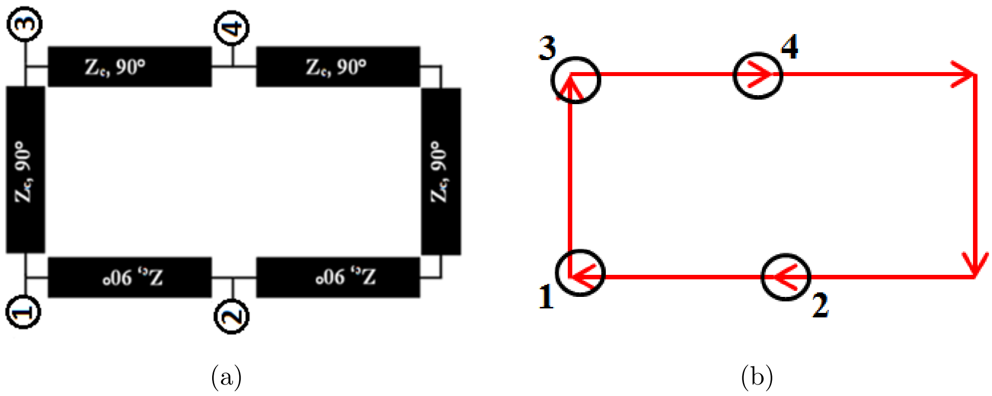


Figure 4: (a) Conventional RRC with six identical quarter wavelength TL, (b) Equivalent RRC with six identical asymmetric TPNs.

TPN based network Fig. 4(b)(calculated using six set of two-port network parameters) to that of the conventional RRC (Fig. 4(a)) and derive an exact synthesis guideline as discussed in the theorems below.

Theorem 2.1 *The asymmetric-TPN based ring network shown in Fig.4(b) will be equivalent to the conventional quarter-wavelength TL based rat-race coupler as in Fig. 4(a),provided the TPN satisfies the following conditions:*

$$a + d = 0 \qquad \text{(3a)} \qquad\qquad b = Z_c \qquad \text{(3b)}$$

where, $\begin{bmatrix} a & jb \\ jc & d \end{bmatrix}$ *are the ABCD parameters of the elementary TPN and* Z_c *being the characteristic impedance of the* $\lambda/4$ *TL.*

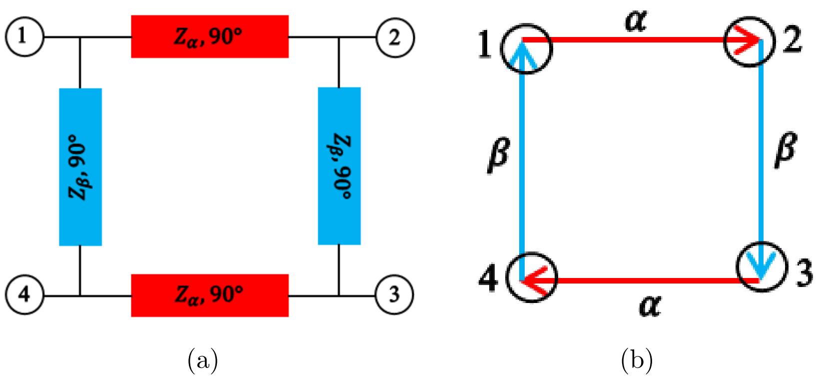


Figure 5: (a) Conventional BLC with two set of identical quarter wavelength TL, (b) Equivalent BLC with two set identical asymmetric TPNs.

2.3 Branch Line Coupler (BLC)

Following the same philosophy as in the case of a rat-race coupler, we extend the idea to the synthesis of asymmetric structure (any of the given building blocks as in Fig.3) based branch-line coupler using the following Theorem.

Theorem 2.2 *The asymmetric-TPN based ring network shown in Fig. 5(b) will be equivalent to conventional quarter-wavelength TL based branch-line coupler as in Fig. 5(a), provided the TPN satisfies the following conditions:*

$$a_\alpha Z_\beta + d_\beta Z_\alpha = 0 \quad (4a) \qquad a_\beta Z_\alpha + d_\alpha Z_\beta = 0 \quad (4b)$$

$$b_\alpha = Z_\alpha \quad (4c) \qquad b_\beta = Z_\beta \quad (4d)$$

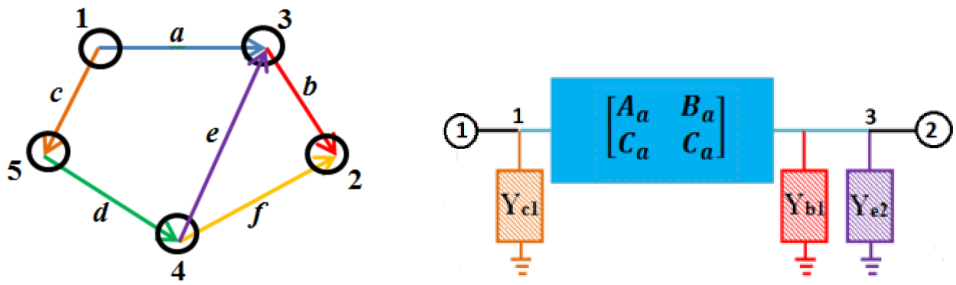
where, $\begin{bmatrix} a_\alpha & jb_\alpha \\ jc_\alpha & d_\alpha \end{bmatrix}$ *and* $\begin{bmatrix} a_\beta & jb_\beta \\ jc_\beta & d_\beta \end{bmatrix}$ *are the ABCD parameters of the elementary TPN* α *and* β. Z_α *and* Z_β *are the characteristic impedance of the* $\lambda/4$ *TL.*

3 Port-Decomposition Technique and its Applications

3.1 Multi-port Network Synthesis using Port-Decomposition

The systematic way to derive the design equations for a complicated multiport (N-port) network with known scattering matrix S_N of dimension $N \times N$ is as follows:

Step-i Choose appropriate topology of the N-port network consisting of n two-port building blocks(TPN) with their ABCD parameters $\begin{bmatrix} A_\kappa & B_\kappa \\ C_\kappa & D_\kappa \end{bmatrix}$, κ belonging to the set $\kappa \in \{\kappa_1, \kappa_2, \ldots, \kappa_n\}$, where $n \geq (N-1)$ such that there always exists a path between any two pair of nodes.



(a) (b)

Figure 6: (a) Five port network with arbitrary topology (b) Reduced two-port network (2PN) of (a) when port-2, 4 and 5 are short circuited.

As an example we choose the 5-port network shown in Fig. 6(a), consisting of interconnections of 6 TPNs belonging to the set $\{a, b, c, d, e, f\}$.

Step-ii Chose two arbitrary ports among the N-ports and decompose the rest of the $(N-2)$ ports by short circuiting them, such that after decomposition the reduced network is *feasible*. In other words, there exists a transmission between the ports.

For the previous example one can choose port-1 and port-3 as a part of the reduced set and decompose port-2, port-4 and port-5 by short circuiting them.

Step-iii Calculate the reduced set scattering parameters S'_2 of the decomposed network from the known S_N with the load reflection coefficient at the N-2 ports being $\Gamma_L = -I_{N-2}$ using [15, (8)]. Here, I_K is the identity matrix of dimension K.

For the given example, the 5-port network S'_2 can be calculated using [15, (8)], where, S'_2 denotes the calculated scattering parameter between ports 1 and 2. Here 1 and 2' represents port-1 and port-3 of the original 5-port network as in Fig. 6(a). Note that the calculated S-parameters are known quantities.

Step-iv Convert the 2-port scattering parameters (calculated in Step-iii) into a suitable two port representation (i.e., ABCD parameter).

Step-v Determine the reduced 2-port network (2PN) by removing those TPN blocks with both its ports short-circuited and converting TPN blocks whose single ports are short circuited into equivalent shunt admittance using

$$Y_{\kappa 1} = \frac{D_\kappa}{B_\kappa} \qquad (5a) \qquad\qquad Y_{\kappa 2} = \frac{A_\kappa}{B_\kappa}. \qquad (5b)$$

Here, the subscript-1(2) refers to the case where port-2(1) of the TPN has been short-circuited. Determine the ABCD parameters of the 2PN in terms of the unknown parameters belonging to the set κ.

In the example, the TPNs d and f are removed and b, c and e are converted into the equivalent shunt admittance as discussed before. The ABCD parameters of the reduced 2-port network shown in Fig. 6(b) is given as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_{c1} & 1 \end{bmatrix} \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_{b1} + Y_{e2} & 1 \end{bmatrix} \tag{6}$$

Note that these ABCD parameters are function of the unknown network parameters of the building blocks belonging to the set $\{a,\ b,\ c,\ e\}$

Step-vi Equating the ABCD parameters derived in Step-v to those calculated in Step-iv, one can obtain a subset of the required design equations.

Step-vii Repeat Steps-ii to vi with a different choice of *2*-ports as part of 2PN, until a unique and complete set of design equations is obtained.

Four theorems have been established in the next sub-section illustrating the above synthesis procedure.

3.2 Application of the Proposed Algorithm

In this section, we discuss four theorems on the design equations of a generalized RRC and BLC, a five port network with equal power division at the output ports, using general asymmetric TPNs as building blocks and a generalized phase-difference coupler (PDC) consisting of coupled lines. The last example illustrates a case which consists of elementary building blocks as a combination of two-port network and four-port networks (FPN) due to presence of coupled lines. The theorems have been established based on the above algorithm for multi-port network synthesis.The theorems are as follows:

Theorem 3.1 *A four port network arranged in a directed ring topology shown in Fig. 7(a) is electrically identical with a generalized rat-race coupler having arbitrary transmission phase (φ) and unequal power division (1: n) at the output ports, with its scattering parameters given as*

$$S_{GRRC} = e^{-j\varphi} \begin{bmatrix} 0 & \tau_1 & \tau_2 & 0 \\ \tau_1 & 0 & 0 & -\tau_2 \\ \tau_2 & 0 & 0 & \tau_1 \\ 0 & -\tau_2 & \tau_1 & 0 \end{bmatrix} \tag{7}$$

where $\tau_1 = \frac{1}{\sqrt{(1+n)}}$, $\tau_2 = \frac{\sqrt{n}}{\sqrt{(1+n)}}$; *provided the elements of the four-port ring network satisfies*

$$\sqrt{n} a_\alpha + d_\beta = \sqrt{n} d_\alpha + a_\beta = \sqrt{(n+1)} \cos\varphi \tag{8a}$$

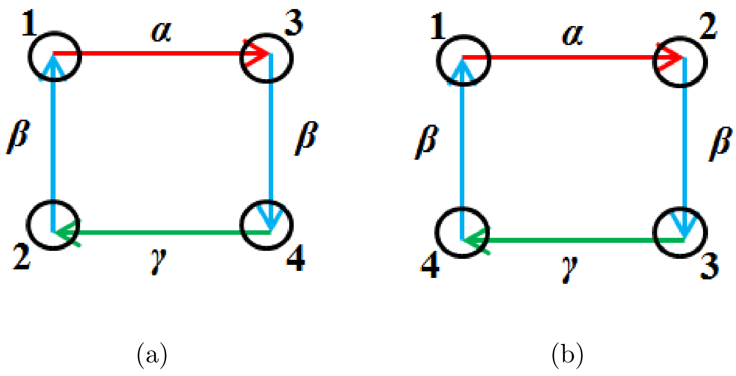


Figure 7: (a) Generalized RRC with asymmetric TPNs as its building block; (b) Generalized BLC with asymmetric TPN as its building block

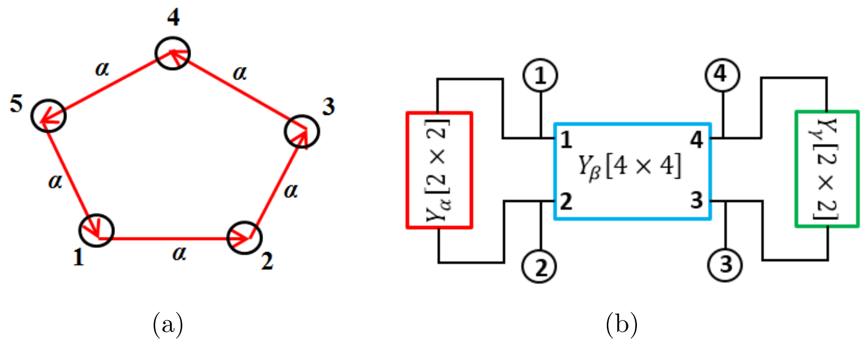


Figure 8: (a) Topology of a TPN based five port equal power division network.; (b) Topology of a symmetric-FPN and TPN based PDC.

$$b_\alpha = \sqrt{\frac{(n+1)}{n}} Z_0 \sin\varphi \qquad \text{(8b)} \qquad\qquad b_\beta = \sqrt{(n+1)} Z_0 \sin\varphi \qquad \text{(8c)}$$

$$\begin{bmatrix} a_\gamma & b_\gamma \\ c_\gamma & d_\gamma \end{bmatrix} = -\begin{bmatrix} a_\alpha & b_\alpha \\ c_\alpha & d_\alpha \end{bmatrix}. \tag{8d}$$

Theorem 3.2 *A four port network arranged in a directed ring topology shown in Fig. 7(b) is electrically identical with a generalized branch-line coupler having arbitrary transmission phase (φ) and unequal power division $(1:n)$ at the output ports, with its scattering parameters given as*

$$S_{GBLC} = \begin{bmatrix} 0 & \tau_1 e^{-j\varphi} & -\tau_2 & 0 \\ \tau_1 e^{-j\varphi} & 0 & 0 & -\tau_2 \\ -\tau_2 & 0 & 0 & -\tau_1 e^{j\varphi} \\ 0 & -\tau_2 & -\tau_1 e^{j\varphi} & 0 \end{bmatrix} \tag{9}$$

with $\tau_1 = \frac{1}{\sqrt{(1+n)}}$, $\tau_2 = \frac{\sqrt{n}}{\sqrt{(1+n)}}$; *provided the elements of the four-port ring network satisfies*

$$\sqrt{(n+1)}a_\alpha + \sqrt{n}d_\beta = \sqrt{(n+1)}d_\alpha + \sqrt{n}a_\beta = \cos\varphi \tag{10a}$$

$$\sqrt{(n+1)}a_\gamma + \sqrt{n}d_\beta = \sqrt{(n+1)}d_\gamma + \sqrt{n}a_\beta = -\cos\varphi \tag{10b}$$

$$b_\alpha = b_\gamma = \frac{1}{\sqrt{(n+1)}} Z_0 \sin\varphi \qquad (10c) \qquad\qquad b_\beta = \frac{1}{\sqrt{n}} Z_0 \sin\varphi \qquad (10d)$$

Theorem 3.3 *A five port network arranged in a directed ring topology shown in Fig. 8(a) is electrically identical with a conventional equal power division five port network with its scattering parameters given as*

$$S_{5PN} = \frac{1}{2}\begin{bmatrix} \sigma & \tau & \mu & \mu & \tau \\ \tau & \sigma & \tau & \mu & \mu \\ \mu & \tau & \sigma & \tau & \mu \\ \mu & \mu & \tau & \sigma & \tau \\ \tau & \mu & \mu & \tau & \sigma \end{bmatrix} \qquad (11)$$

with $\sigma = 0$, $\tau = e^{-j\frac{\pi}{3}}$, $\mu = -1$; *provided the elements of the five-port ring network satisfies*

$$a_\alpha + d_\alpha = \frac{1}{2} \qquad (12a) \qquad\qquad b_\alpha = \frac{\sqrt{3}}{2} Z_0 \qquad (12b)$$

Theorem 3.4 *A four port network topology shown in Fig. 8(b) consist of a symmetric FPN (*β*) and two symmetric TPN (*α *and* γ*), is electrically identical with a generalized phase difference coupler having arbitrary transmission phase (*φ*), phase difference (*ψ*) and unequal power division* $(1:n)$ *at the output ports, with its scattering parameters given as*

$$S_{PDC} = e^{-j\varphi}\begin{bmatrix} 0 & \tau_1 & \tau_2 e^{-j\psi} & 0 \\ \tau_1 & 0 & 0 & \tau_2 e^{-j\psi} \\ \tau_2 e^{-j\psi} & 0 & 0 & -\tau_1 e^{-j2\psi} \\ 0 & \tau_2 e^{-j\psi} & -\tau_1 e^{-j2\psi} & 0 \end{bmatrix} \qquad (13)$$

with, $\tau_1 = \frac{1}{\sqrt{(1+n)}}$, $\tau_2 = \frac{\sqrt{n}}{\sqrt{(1+n)}}$; *provided the TPN and FPN elements of the network satisfies*

$$y_{11\alpha} + y_{11\beta} = \frac{-(n+1)\sin 2(\varphi+\psi) + \sin 2\psi}{n + \cos 2\psi - (n+1)\cos 2(\varphi+\psi)} \qquad (14a)$$

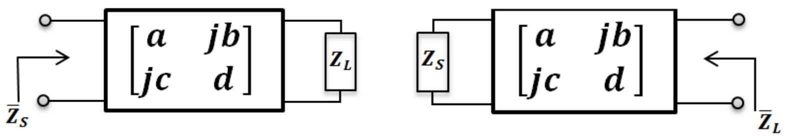
$$y_{21\alpha} + y_{21\beta} = 2\sqrt{(1+n)}\frac{\sin(\varphi+2\psi)}{n + \cos 2\psi - (n+1)\cos 2(\varphi+\psi)} \qquad (14b)$$

$$y_{11\gamma} + y_{11\beta} = \frac{-(n+1)\sin 2(\varphi+\psi) - \sin 2\psi}{n + \cos 2\psi - (n+1)\cos 2(\varphi+\psi)} \qquad (14c)$$

$$y_{21\gamma} + y_{21\beta} = 2\sqrt{(1+n)}\frac{\sin\varphi}{n + \cos 2\psi - (n+1)\cos 2(\varphi+\psi)} \qquad (14d)$$

$$y_{31\beta} = 2\sqrt{n(1+n)}\frac{\sin(\varphi+\psi)}{n + \cos 2\psi - (n+1)\cos 2(\varphi+\psi)} \qquad (14e)$$

$$y_{41\beta} = -2\sqrt{n}\frac{\sin\psi}{n + \cos 2\psi - (n+1)\cos 2(\varphi+\psi)} \qquad (14f)$$



(a) (b)

Figure 9: (a) A two-port lossless passive reciprocal network transforming a load impedance $Z_L = R_L + jX_L$ into complex conjugate of the source impedance $\bar{Z}_S = R_S - jX_S$; (b) The same network transforming the source impedance $Z_S = R_S + jX_S$ into complex conjugate of the load impedance $\bar{Z}_L = R_L - jX_L$.

where

$$Y_{\alpha,\gamma} = j \begin{bmatrix} y_{11\alpha,\gamma} & y_{21\alpha,\gamma} \\ y_{21\alpha,\gamma} & y_{11\alpha,\gamma} \end{bmatrix} \qquad Y_\beta = j \begin{bmatrix} y_{11\beta} & y_{21\beta} & y_{31\beta} & y_{41\beta} \\ y_{21\beta} & y_{11\beta} & y_{41\beta} & y_{31\beta} \\ y_{31\beta} & y_{41\beta} & y_{11\beta} & y_{21\beta} \\ y_{41\beta} & y_{31\beta} & y_{21\beta} & y_{11\beta} \end{bmatrix}.$$

4 Matching Network

An impedance matching network transforms the load impedance ($Z_L = R_L + jX_L$) into complex conjugate of source impedance ($\bar{Z}_S = R_S - jX_S$) [16] or in the other words transforms the source impedance ($Z_S = R_S + jX_S$) into complex conjugate of the load impedance ($\bar{Z}_L = R_L - jX_L$) as shown in Fig. 9. There are possibly infinite lossless reciprocal two-port network (TPN) parameter set which satisfies the above condition. An unique set of TPN parameters are obtained when the transmission phase-shift is taken into account. The network parameters of an IMN can be established using the following theorem:

Theorem 4.1 *A complex source impedance ($Z_S = R_S + jX_S$) and a load impedance ($Z_L = R_L + jX_L$) connected to port-1 and port-2 of a lossless reciprocal two-port network respectively, provides a maximum power transfer from port-1 to port-2 with a transmission phase-shift of φ, when the network satisfies:*

$$a = h\,(X_s \sin\varphi + R_S \cos\varphi) \tag{15a}$$

$$b = h\,((R_S R_L - X_S X_L)\sin\varphi - (R_S X_L + X_S R_L)\cos\varphi) \tag{15b}$$

$$c = h \sin\varphi \quad (15c) \qquad d = h\,(X_L \sin\varphi + R_L \cos\varphi) \quad (15d)$$

where $h = 1/\sqrt{R_S R_L}$ *and* $\begin{bmatrix} a & jb \\ jc & d \end{bmatrix}$ *is the ABCD matrix of the IMN.*

5 Conclusions and Contributions Made by the Scholar

A conceptually correct way of synthesizing conventional equal power division rat-race and branch-line coupler using asymmetric-two-port network have been proposed. The synthesis equations are laid down in form of two theorems. A generalized multi-port network synthesis algorithm has been proposed based on a novel philosophy of port-decomposition. This algorithm can be applied to any type of multi-port networks viz. symmetric or asymmetric, perfectly matched or unmatched, lossy or lossless etc. Concept of true transmission phase and its implications has been introduced. Four theorems pertaining to multi-port network synthesis have been postulated. The theorems provides the conditions for realization of a generalized rat-race and branch-line couplers, a five-port network using such two-port building blocks and an arbitrary phase difference coupler consisting of coupled lines. The two-port lossless ABCD parameters of a matching circuit with desired transmission phase has been derived and its importance has been discussed, which matches an arbitrary complex load to a different complex source impedance.

6 Contents of the Thesis

Chapter 1 of the thesis contains brief introduction and literature review of a multi-port network. In Chapter 2, design equations have been obtained for replacing each quarter wave-length transmission line of a rat-race and a branch-line coupler, by a generalized two-port network (symmetric or asymmetric). A systematic approach (algorithm) to synthesize multi-port (N-port) network with desired characteristics, for a given network topology, has been proposed in Chapter 3. Utilizing this algorithm four theorems pertaining to multi-port network synthesis have been proposed. Chapter 4 deals with synthesis of impedance matching network incorporating the transmission phase-shift. Finally, Chapter 5 concludes the thesis and sketches possible scope of future works.

7 List of Publications Based on the Research Work

Journals

1. R. Sinha, A. De, and S. Sanyal, "A Theorem on Asymmetric Structure based Rat-Race Coupler," *IEEE Microw. Wireless Compon. Lett.*, vol. 25, no. 3, pp. 145-147, Mar. 2015.

2. R. Sinha and A. De, "Synthesis of multiport networks using port decomposition technique and its applications,"*IEEE Trans. Microw. Theory Techn.*, vol. 64, no. 4, pp. 1228-1244, Apr. 2016.

3. R. Sinha and A. De, "Theory on Matching Network in Viewpoint of Transmission Phase-shift," *IEEE Trans. Microw. Theory Techn.*, doi: 10.1109/TMTT.2016.2558645.

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