SYNOPSIS

The main task in an identification problem is to process the input-output data from an active operating record over a given interval and to determine the parameters in a model of chosen form. For continuous models, there are two basically different approaches. One method, frequently employed in identification, is to treat the process signals as functions in the ordinary sense. There exists another method, not fully exploited so far, in which the process signals are treated as distributions or generalised functions in the manner originally conveived and established by Dirac and Schwartz respectively. The latter characterisation of process signals is superior due to its unlimited differentiability. The present work takes up this approach and develops general algorithms for parameter identification in continuous systems.

For the purpose of the present work a signal $f(t)$, $t \in (0, t_o)$, is treated as a distribution or a generalised function and expanded about a time t_o in the following exponentially weighted series as originally suggested by Fairman and Shen [F1, F2].

$$f(t) = \sum_{k=o}^{\infty} M_k \left\{ f(t) \right\} e^{-\lambda (t-t_o)} \delta^{(k)}(t-t_o) ,$$

where $\delta^{(k)}(t-t_o)$ is the k-th generalised time derivative of an impulse distribution occurring at $t = t_o$, and

$$M_k \left\{ f(t) \right\} \triangleq f_k = \int_0^{t_o} f(t)\, p_k(t_o - t)\, dt ,$$

$$p_k(t) = \frac{t^k}{k!} e^{-\lambda t}$$; a k-th order Poisson pulse function and λ is a positive real number. f_k is called the k-th Poisson moment functional (PMF) of $f(t)$ about t_0. It can be viewed as the output at $t = t_0$, of the (k+1)-th stage of a cascaded filter, each element of which has a transfer function $\frac{1}{(s+\lambda)}$. Some attractive features of the PMF characterisation are:

(a) The PMF transformation converts a process differential equation into an algebraic equation without any approximation. This is not so in other methods such as discretisation of continuous models, numerical approximation method , Walsh function methods etc. In these methods the process of reduction from continuous calculus to discrete algebra involves some approximation;

(b) Noise accentuating derivative operations on process data are efficiently avoided;

(c) The integrals required in the determination of PMF's need not be computed off line. They can be "measured" physically as well-behaved output signals at t_0 of the various stages of a Poisson filter chain excited by $f(t)$;

(d) The PMF characterisation has unlimited differentiability; and

(e) The PMF's are inherently immune to zero mean additive noise to some extent.

The basis of the present method may be traced back to

Shinbrot's method function [S2] approach. In order to avoid undesirable derivative operations on a process signal $f(t)$, Shinbrot multiplies $f(t)$ with a function in analytical form and integrates the result. In doing so the need to perform derivative operation on $f(t)$ is transferred to the chosen function and its derivative in the known analytical form is used. Shinbrot's method functions do not suggest a regular pattern of derivative relations for easy implementation in models of general forms. The idea of method functions is extended to the case of two dimensions by Perdreauville and Goodson [P1] in the identification of distributed parameter systems (DPS) by two dimensional method functions termed in the context as "modulating functions". Here again, no regular pattern of partial derivative relations is indicated for easy implementation in a general case. On the other hand, the PMF characterisation suggests a regular pattern of derivative relations leading to a systematic procedure for dynamic system models in a general form. The PMF approach has been illustrated in identifying certain time varying and nonlinear models [F1, F3]. It has been extended to handle distributed parameter models [F2]. The related partial differential equations (PDE) are first reduced to a set of ordinary differential equations (ODE) by numerically approximating space derivative terms (by method of lines). Then by applying PMF technique to these ODE in time, the model parameters are determined. Diamessis's repeated integration method [D1, D2, D3] for lumped system identification may be viewed as a special case with $\lambda = 0$. Prasada Rao and

Sivakumar placed the PMF technique in a general framework [P3] of available techniques and suggested an iterative method [P5] of identifying the parameters in models containing unknown time delays.

Although the usefulness of the PMF approach has been well established, its potential has not been fully exploited so far in respect of the following:

(i) The original relations of the PMF's of the derivatives of a function are not general as they do not include the effect of initial conditions of the function. Fairman and Shen suggest the use of large λ to reduce the effect of initial conditions. This method while reducing the effect of initial conditions increases the influence of noise present in the signals;

(ii) The identification algorithms lack a general format. A convenient format is desirable for certain frequently encountered models of general and standard forms. It would be desirable if the format also allows inclusion of the case of unknown time delays;

(iii) There is no attempt so far on the study of conditions of identifiability with respect to the PMF approach;

(iv) The PMF technique remains to be studied in the case of identification of multi input-multi output (MIMO) systems. Mathew and Fairman [M2] studied the special case of $\lambda = 0$ for identifying transfer function matrix of MIMO systems. The problem of identification of MIMO systems raises important questions relating to conditions of identifiability;

(v) The existing PMF technique of handling distributed parameter systems is a hybrid one. PMF's are employed in time domain alone while a multi-dimensional function is treated as an ordinary function along the spatial coordinates; and

(vi) The performance of the PMF method, in general, under noisy situations has not been studied.

The thesis addresses itself to these important aspects of system identification by the PMF approach and the following are the salient contributions of the present work;

I. The formulae for PMF's of the derivatives of a function are now generalised to include the effect of initial conditions [S11]. Consequently, the choice of λ becomes free. An important advantage of this generalisation, in practice, is that the PMF method can be employed in the case of process data on an arbitrary (but active) period of operating record.

A detailed study of the calculus of signals in the realm of PMF's leads to an elegant mathematical formulation of relations in a format [S10] which has similarities with the state space method of system description. Representation of PMF relations in this format is advantageous in that it provides us with a clear picture of the correspondence between the equations of dynamical systems in the time-and PMF-domains. The present format of PMF relations lends itself to systematic development of general programme packages (Chapter I).

II. A general algorithm for parameter identification in linear lumped time varying continuous time single input single output (SISO) systems has been developed [S13] in the format

$$\Phi U = C ,$$

where the information matrix Φ and vector C are derived from the PMF's of the process signals and U is a vector of unknown parameters including initial conditions. The case of parameter identification in time invariant models emerges as a special case of the general algorithm under certain simplifications while the format remains the same.

The techniques of Prasada Rao and Sivakumar [P5] for identifying time delay systems needs a suitable step to initiate iteration. The thesis presents a direct algorithm to arrive at such a step. If the delay is small the results of this direct algorithm are themselves accurate enough needing no further iteration.

Conditions of identifiability on the context of PMF approach are discussed for the first time in this work. Identifiability is related to "Poisson Suitability" of input signals as defined and detailed in the thesis. Poisson suitability conditions clearly indicate the class of signals that singularizes the information matrix Φ in the identification algorithm.

Methods of reducing the problem dimension in large systems by deliberately choosing a known class of Poisson unsuitable signals, are discussed (Chapter II).

III. The algorithm developed for SISO systems has been extended straightaway to the case of transfer function matrix identification in MIMO system. A study of the conditions of identifiability in MIMO problems, presented in the thesis, gives rise to two important conditions. One is related to the Poisson suitability of the input signal set, close in spirit to the L-suitability conditions of Mathew and Fairman [M 2]. The other, overlooked by the earlier researchers, is related to an additional condition termed as "Pair Suitability" of input set with each output signal (Chapter III).

IV. As the existing PMF method for DPS is hybrid in nature, the advantage of PMF characterisation are not fully utilised. In view of this situation the concep of multi-dimensional PMF's is introduced in this thesis. Multi-dimensional PMF's and derivative relationships have been systematically established [S12]. Algorithms for parameter identification in DPS are presented and extensively illustrated. The multi-dimensional PMF approach is found to be superior to the well known technique of Collins and Khatri [C4], particularly in the presence of noise (Chapter IV).

The thesis contains several illustrative examples at various stages. It is interesting to note that the PMF's are sufficiently immune to zero mean additive noise and so are the identification algorithms based on these. This feature is illustrated in the thesis.

It is the author's contention that the present work removes the existing limitations of the PMF approach placing it on firm foundation with standrdised formats for general application.