SYNOPSIS

A fundamental limitation of linear time-invariant (LTI) feedback controllers is that they can not alter the plant zeros [18]. As a consequence, they can not provide satisfactory control (e.g. gain margin compensation [30], strong and simultaneous stabilization [10], disturbance rejection [25]) in some situations, particularly when the plant has non-minimum-phase (NMP) zeros. This shortcoming of LTI controllers has led researchers to investigate if periodic controllers would improve matters, and this has become a topic of considerable interest in current literature [1], [3], [4], [7], [8], [12], [19]-[21], [27], [28].

The periodic compensation problem may be divided under two heads: (a) continuous time, (b) discrete time. Analysis of periodic coefficient differential equations being a difficult task, the continuous-time problem has received only limited attention in literature. In fact, the work of Lee, Meerkov and Runolfsson [21] appears to be the only one that deals with this problem in some depth, using the averaging principle as the method of analysis which, though, is valid only at very high frequencies. (A better analytical tool to deal with such systems has recently been developed in [8]). The important contributions of [21] are that (a) it considers the controller in a minimally realizable form, and (b) it introduces the concept of zero-placement. Discrete-time periodic systems, on the other hand, have received considerably more attention in literature [4], [7], [12], [19], [20], [27], [28], the reason for this perhaps being the seminal work of Khargonekar, Poolla and Tannenbaum [20], in which a convenient framework for the analysis of such systems has been developed.

A study of the works pertaining to discrete-time periodic controllers shows that the following definite conclusions regarding their capabilities have been reached:

a) They can alter the system zeros [7], [12].

b) They can provide infinite gain margin (GM) compensation to unstable bicausal plants having NMP zeros [7], [12], [20].

c) They, however, can not better LTI controllers in regard of uniform disturbance rejection [3], [20], [27].

The same study, however, reveals certain incompletenesses in the understanding of such controllers as well, especially in the aspects of their

i) realizations,

ii) effectiveness in providing GM compensation to strictly proper plants,

iii) simultaneous stabilization capability, and

iv) capability to reject disturbances at even/(odd) instants of time.

We elaborate on these points further, restricting attention to SISO systems :

i) It follows from [20] that a causal, linear, SISO, M-periodic input-output map can be isomorphically represented as an MxM LTI transfer matrix T(z) with T(∞) lower triangular. This result is very convenient, for it allows one to treat a SISO periodic problem as a MIMO-LTI one. In fact, most of the works in literature assume the periodic controller in a MIMO-LTI form and then proceed to obtain the elements of the same so as to achieve the design goal. Now, although such a T(z) may obviously be realized element by element using a set of asynchronous samplers that link each one of the elementary transfer functions to the input-output lines at the correct time instants, no clear understanding exists in literature regarding their minimal realizations. In other words, a general yet minimal representation of causal periodic controllers is yet to be obtained in literature. This problem is worth serious effort, for the design of periodic systems based on such a controller configuration would not only involve a minimum number of unknowns but would also ensure causality automatically. (Note that the MIMO-LTI design approach does not guarantee causality synthetically. In fact constrained optimization techniques are employed to ensure the same [4]). In this context the controller structure of Das and Rajagopalan [7] may be noted. Although its capability for zero-placement and GM compensation has been demonstrated, its input-output representation in a transfer function form has not been obtained in [7]. Hence its full potential remains to be tapped.

ii) While the maximum GM obtainable for an unstable plant having NMP zeros using LTI controllers is a finite number whose value can be calculated following well known procedures [17], [30], it has been shown in literature [7] that 2-periodic controllers can provide infinite GM compensation to such plants if they are bicausal in nature. So far as strictly proper plants are concerned, however, although it has been observed [7], [20], through numerical examples that periodic controllers might provide better GM compensation than LTI ones, the problem of their optimum GM compensation using periodic controllers still remains unsolved. Moreover, in literature two approaches have been used to provide GM compensation using periodic controllers. These are : (a) the zero-placement approach [7], and (b) the factorization approach [20]. (It may be noted that both these approaches consist of shaping the closed-loop characteristic equation in certain forms, and although the latter is more general, the former is important since it maintains system linearity.). So, for a complete understanding of the problem it becomes necessary that the optimum GM obtainable for strictly proper plants using periodic controllers through either of these approaches be investigated.

iii) While it is not always possible to stabilize two arbitrary plants simultaneously using a single LTI controller [10], [14], [15], [34], it has been claimed in [20] that it is possible to do so using 2-periodic controllers. No explanation, either analytical or numerical, has, however, been presented in [20] to substantiate this claim. Consequently, this remains a open topic for investigation. In this context, it would also be interesting to investigate if, in general, an M-periodic controller would be able to provide simultaneous stabilization to an M-tuple of plants. (It may be noted that [20] does describe a procedure for obtaining periodic controllers for simultaneous stabilization of K-tuple of plants. This result, however, is of theoretical interest only, for the procedure requires the evaluation of deadbeat controllers for each one of the K-tuple of plants and, moreover, results in a controller that has a very high period).

iv) It has been established in literature [3], [27] that, regardless of the norm used, periodic controllers can not do better than LTI ones in the respect of disturbance rejection in LTI plants. Now, since LTI controllers can not provide uniform (unweighted) disturbance rejection for NMP plants [10], [25], it follows that periodic controllers can not do so as well. One may, however, be interested in disturbance rejection not at all but at some regular (say even) instants of sampling. The question then is : Will periodic controllers be able to provide superior disturbance rejection at such instants, especially for NMP plants? This problem, moreover, arises naturally from the point of view of multirate systems in which the input is provided at some regular intervals of time and the output taken at some other. Now, although multirate discrete-time systems are, by themselves, a topic of considerable interest in literature [16], [23], [24], [26], the above question, as it is, does not appear to have been considered so far.

This thesis aims to provide a better understanding of the working and capabilities of SISO, discrete-time, periodic controllers by filling up the above gaps. Its main contributions are as follows:

a) It shows that if the MIMO-LTI equivalent, $T(z)$, of a SISO-periodic map is represented as $T(z)=N(z)/d(z)$ where $N(z)$ is a polynomial matrix and $d(z)$ is a polynomial of degree m(say), then, in addition to $T(\infty)$ being lower triangular (as shown in [20]), $N(0)$ will be upper triangular. Consequently, it obtains a 2m-order realization for $T(z)$.

b) Noticing, however, that the above 2m-order realization does not utilize all its degrees of freedom, it then considers a maximum-degrees-of-freedom m-order controller structure and obtains its equivalent LTI transfer matrix.

c) Next, it obtains the closed-loop transfer function and the characteristic equation of systems having an LTI plant and a periodic controller, and shows that so far as characteristic equation manipulations are concerned it is enough to consider the controller as in (b).

d) Regarding GM compensation of strictly proper plants it shows that (i) if the zero-placement approach is used then the optimum GM would be obtained for a controller periodicity, M, that is a prime number $\leq (n-r)$ where n and r are, respectively, the number of poles and zeros of the plant, and (ii) in general, the optimum GM is obtained for an M lying in the range $(n+1) \geq M \geq (n-r)$.

e) It clearly shows that while, generically, an M-periodic controller can simultaneously stabilize (in fact, achieve pole placement of) M different plants, M arbitrary.

f) Lastly, it shows that 2-periodic controllers can be used to reject disturbances at the even/(odd) instant outputs. In addition, steady-state step command tracking at the odd instants can also be achieved.

The thesis is divided into six chapters, the development being along the following lines:

Chapter 1: This chapter reviews critically the results available in literature on discrete-time periodic systems. First the result of [20] regarding the lifting of periodic systems into equivalent (larger) LTI forms is presented. This result permits one to assume the controller to be in a MIMO-LTI form and then design its elements using LTI methodologies. This form of the controller, however, is not minimal from the point of view of realization and is not automatically causal. Next, we present the controller configuration of [7] and the method developed therein to analyze the stability of the composite system by obtaining the characteristic roots of the difference equation that has periodic coefficients. Although the controller of [7] is in a minimal form and, moreover, can be conveniently represented in the form of a periodic coefficient transfer function, its equivalent LTI representation has not been obtained. Next, the GM compensation aspect is considered. In this context, first the methods available for finding the maximum GM obtainable for a given plant using LTI compensators are presented briefly, and then the zero-placement and the factorization approaches available for compensation using periodic controllers are reviewed. It is seen that periodic controllers are capable of providing infinite GM compensation to bicausal plants. The same, however, is not true for strictly proper plants, and the question of optimum GM compensation of such plants using periodic controllers is still open. Finally, this chapter presents the observations of literature regarding simultaneous stabilization and disturbance rejection using periodic controllers, and notes that (a) the simultaneous stabilization capability is not clearly understood, and (b) even though periodic controllers can not provide better disturbance rejection that LTI ones at all time instants, the possibility of their providing the same at alternate instants has not been explored.

Chapter 2 : Seeking to obtain a maximum-degrees-of-freedom configuration of periodic controllers, this chapter first shows that if the representation of a general M-periodic controller is sought in the common denominator form $T(z)=N(z)/d(z)$ where $N(z)$ is an MxM polynomial matrix and $d(z)$ a polynomial of degree m, then $N(0)$ must be upper triangular (besides $T(\infty)$ being lower triangular). This result leads naturally to a 2m-order SISO controller-canonical form [18] realization (with periodic gains). Moreover, in order that the parameter values of this realization corresponding to a given $N(z)$ can be ascertained, a result that lifts SISO periodic coefficient transfer functions to MIMO-LTI forms has been developed. (It may be pointed out that in literature periodic systems have mostly been visualized either in the form of input-output maps or as MIMO-LTI transfer matrices, and not in the form of SISO transfer functions with periodic coefficients). The 2m-order realization, however, is found not to utilize all its degrees of freedom. Consequently, a controller-canonical form maximum-degrees-of-freedom mth order structure for the controller (as considered in [7]) is considered and its equivalent LTI transfer matrix obtained. Some special cases of this controller, name by, the 1-input

M-output and the M-input 1-output cases, are then brought out. It is seen that these special cases can be realized in a simple fashion using only SISO-LTI transfer functions preceded or followed by an MT sampler.

Chapter 3 : This chapter obtains the closed-loop transfer functions of the LTI plant-periodic controller combination for the general and the special controller configurations mentioned in the preceding chapter. Special attention is paid to the system characteristic equations corresponding to these configurations leading to the conclusions that (a) 1-input M-output and the M-input 1-output controllers automatically yield zero-placement type of compensation, and (b) so far as shaping the characteristic equation is concerned, the mth order controller of [7] is as powerful as the Mm-order MIMO-LTI one. In this context, the zero-placement aspect is also discussed in more detail.

Chapter 4 : This chapter considers the problem of optimum GM compensation of strictly proper plants using periodic controllers via (a) the zero-placement, and (b) the general factorization approaches, both of which, as noted earlier, involves shaping the characteristic equation in certain fashions. The conclusions reached in this regard have been noted earlier. Suitable examples are also considered to illustrate the results.

Chapter 5 : Based on a study of the closed-loop characteristic equation, this chapter shows that it is possible to place the poles of (and hence stabilize) M arbitrary plants simultaneously using a single M-periodic controller.

Chapter 6 : The problem of rejection of disturbances (occurring at all instants of time) at the even/(odd) instant outputs of NMP, bicausal plants is considered in this chapter. It is shown that such disturbance rejection, over the whole of the frequency range, can be achieved by a 2-periodic controller that is a "2-periodic inverse" of the plant. The aspect of evaluation of such 2-periodic inverses is also given due attention. Now, while such controllers would automatically ensure command tracking at the even/(odd) instants, it is shown that it is also possible to extend the same to achieve steady-state step command tracking at the odd/(even) instants as well.

In the end we would like to point out some possible extensions of the present work that are relevant and should prove to be interesting. First, and obvious, is the extension of the present result to MIMO systems. In particular, the problem of finding a minimal representation for the MIMO periodic controllers appears to be quite challenging. A generalization of the concept of "2-periodic inverses" should, on the other hand, be a simpler job and rewarding as well. Besides MIMO systems, the other topic which the thesis has kept out of its purview is that of robust stabilization in the presence of structured/unstructured plant uncertainties. (Note that the GM problem, which has been considered in more detail, is a special case of this general robustness problem). This is a topic of ongoing research [3], [4], [19], [27], [28].