SYNOPSIS

Uncertainty pervades substantiallyin several problems of our every day life. Traditionally, for inferring and decision analysis, tools provided by the probability theory were considered adequate for the purpose. However, In recent years, it has become increasingly clear that uncertainty is a multifacet concept in which some of the important facets do not lend themselves to analysis by the standard probability-based methods. One such facet is that of imprecision, which is characterized by the predicates exemplified by *small* , *large*, *nearly*, *likely* etc. The question is how one can express the meaning of propositions involving such predicates through the use of probability based methods. If this is not possible in a probabilistic framework, then how can one employ the information provided by the proposition in question to bear on a decision relating to it.

These are the questions which motivated the development of fuzzy theoretic techniques for dealing with the problems in which uncertainty derives from the fuzzy imprecision, or more generally, from the presence of both fuzzy imprecision and probabilistic uncertainty [1,4]. Further more, it is the latter case which is highly important in the formulation and solution of many significant real-world problems. This is primarily for the reason that in such problems one usually has to base his decision on the information which is both fuzzily imprecise and probabilistically uncertain.

In reliability engineering, this problem is further more complicated by the fact that in many cases, it is even very difficult to get sufficient data. The probability-based methods partially circumvent this problem by employing Bayesian approach with prior distribution. A prior distribution is constructed from the experience gained from the failure characteristics of similar devices working under similar conditions. The inference is then made from the posterior distribution constructed from the prior distribution and data. Therefore, in reliability estimation, risk analysis and prediction problems, a fusion of Bayesian and fuzzy set approach is desirable. Bayesian component of the approach takes care of the wealth of experience of the analyzer and/or operating personnel and mainly the random nature of the failure mechanism, while the fuzzy set approach takes care of the imprecision of the data. In the present thesis, the author makes an humble attempt to use this concept and expects that it will pave the foundation for future work in this area.

The thesis has been organized into 7 chapters, each highlighting a specific aspect of the problem attacked.

In Chapter I, the author provides a brief introduction to uncertainty methods and fuzzy set theory with main emphasis on the concepts that are used later in this thesis.

The author provides several simple ways to treat uncertainties in the data and covers significantly four major areas of reliability engineering, viz ;

i) fault tree analysis,

ii) system reliability estimation and prediction,

iii) life testing and estimation,

iv) markov modelling of maintained and non-mainted system.

The first part of each chapter is devoted to the development of simple probabilistic (Bayesian) method of treating uncertainty, where as in the second part, the same problem is solved using a probabilistic-possibilistic approach assuming that the data is fuzzily imprecise.

Among the several distributions, Weibull distribution, notably, is a multipurpose distribution and it embraces ,as it does by varying the parameters , a wide range of variation of the probabilities. It has been found to be highly satisfactory in describing the life expectancy of components in fatigue induced failures and for computing the time to failure of bearings and CR tubes. It is employed for assessing the reliability of machine parts and units , e.g. of motor vehicles, load handling and other machines. It has also been used for estimating reliability during infancy period. Therefore first part of Chapter II deals with the sequential estimation of the parameters of a Weibull distribution.

One of the most widely used method of estimating the parameters of weibull distribution is least square procedure after the cumulative distribution function transformed with the help of a regression model . A wide range of regression model building techniques such as *forward sections* , backward eliminations, stepwise and all possible subsets regressions, are available today. However all of these techniques are of single shot or batch processing nature, in that the model parameter estimates are calculated based on the entire data set. If new data subsequently becomes available and new parameter estimates are desired the solution procedure must be repeated each time with the new data added to the old data set . Obviously, this mode of solution is not computationally efficient. A Bayesian sequential estimation procedure to accomplish the above task more efficiently than the regression techniques, is presented first and then its application for the estimation of the parameters of Weibull distribution is provided in chapter II. The second part of chapter II discusses some methods, the author has developed for the estimation of parameters of several distributions like exponential, Weibull, normal etc., when the data is imprecise and is represented by fuzzy numbers. The author also discusses the method to find fuzzy confidence intervals for the estimated parameters.

Chapter III deals with the reliability estimation of a system from the uncertain or vague reliabilty of its constituent elements obtained from the tests or field data. In the past, several studies have been done to obtain bounds on the moments of system reliability distribution in which *prior* distributions are assumed for components' reliabilities . The author, utilizing the same technique (method of moments) obtains the *exact* moments very easily and from these moments a prior distribution for system is constructed using the **maximum entropy principle**. From the data obtained, either from the field or from the life tests conducted on the system, a *posterior* distribution is constructed. Further to track the evolution of reliability over time (with the additional information regarding the system failure) a **Kalman filter technique** has been developed .

The second part of Chapter III deals with the system reliability estimation when the reliabilities of the elements are specified as fuzzy numbers between 0 and 1. . The author provides in this chapter, a new and simple approach called *Alpha Cut* approach for the evaluation of reliability of the series-parallel system reliability. Ghosh Chowdhury and Misra [45] proposed a method for non series- parallel network which computes the system reliability in a recursive fashion. The method using *Alpha Cuts* provides simple faster and more accurate method of evaluation than [45]. *To proceed along* the Bayesian method *of* updating using Kalman *filter* technique, a system possibility distribution is converted into a probability distribution by employing the principle of probability-possibility consistency enunciated by Dubois and Prade [4,14]. Finally, the author estimates the reliability of system with multi state elements. The method can also be applied to multi state flow networks.

In Chapter IV, the author deals with Markov models for maintained and non-maintained systems which are characterised by a set of first order differential equations with constant coefficients. The constant coefficients correspond to the functions of repair and failure rates of the components of the system. In real life situations, these coefficients may not actually remain constant and may vary according to the variations in the environmental conditions, type of input etc. One way of handling this situation is to assume that either the coefficicnts are random variable independent of time (in the sense of Bayesian) or are known functions of time with random constants. Making use of the recent developments in the theory of differential equations with random coefficients, the author attempts to solve simple markov models which are applicable to reliability engineering. The author, then, extends this concept to the fuzzy coefficients which may arise in many situations and develop simple solution techniques.

In Chapter V, using the methods developed in Chapter III , for Bayesian and fuzzy set approach, the author analysed event trees under uncertainty. A new method is suggested for ranking the paths in the event trees, when the risk associated with each path is fuzzily imprecise.

In Chapter VI, the author first discusses the problem of uncertainty propagation in fault trees - an active and potential area of research and applications even now. Here the problem which is of our concern is - finding the measure of uncertainty in the top event probability, provided the uncertainties in basic events are known and to ascertain the dependencies in failures. The author provides a new, efficient and an extremely simple algorithm to compute the moments of the top event probability in a fault tree. These moments quantify the uncertainty in failure probabilities propagated from the basic events to the top event. No doubt, this has been a classical problem in reliability engineering and the literature is abundant with several methods such as Monte Carlo simulation , systematic gate-by-gate level combination of random variables , approximate regression analysis , method of moments etc. However, one should realize that it is only the method of moments that provides an accurate answer to the problem. Rushdi [67,69] obtained an expression for the moments using Taylor's series expansion but the expression becomes too complicated particularly in case of higher order moments. Further, no general computer code is available to compute these moments using the Taylor's series expansion . The author approaches this problem in a more direct way and obtains the exact moments from the basic principles of the probability theory. In this way, the moments of any order can be computed without any difficulty and a general algorithm, which is applicable to the fault trees with both two-state and multi state elements has been provided. This algorithm also takes care of dependency of failures.

In literature, we come across several measures of importance [20]. In the calculation of these importance measures, it is assumed that there is no uncertainty regarding the numerical value of the basic event probabilities. Therefore, in the context of uncertainty where unreliability is expressed as a probability distribution (in the sense of Bayesian), previous importance measures must be modified to take in to account the spread of the unreliability ofthe basic events as well. Further, if there is a high amount of uncertainty in the reliability / unreliability of the system , we must also identify the basic events whose uncertainties of probabilities are contribute significantly to the uncertainty of probability of the system. This helps in deciding the components/subsystems for which more data or information is required to reduce the uncertainty of the system failure probability. To this end, Nakashima et.al [64] introduced the concept of variance sensitivity coefficients and the importance measures. They derived expression for these measures using Taylor's series expansion. These expressions became too lengthy, even for a small and simple series and parallel system. Therefore, in chapter VI a new and simple method of computing the sensitivity coefficients and importance measures simultaneously is presented. The method has been generalized to include dependent failures also.

The second part of Chapter VI concerns itself with the fuzzy fault tree analysis. Tanaka [11] and Misra [18,19] were among the first to introduce fuzzy theoretic concepts in a fault tree analysis. The author proposes a new and simple approach which obviates several of the drawbacks of the previous methods available in the literature . Furthermore, the author has developed a general algorithm which accommodates multi state elements in the fault tree . As far as the author is aware , this is the first attempt using fuzzy concepts for the analysis of multi state fault tree. Finally in this chapter author discusses fuzzy importance measures.

Many problems arising in scientific investigations generate data incorporating non-statistical uncertainty. In such instances, a fuzzy axiomatic structure of dealing with such problems usually increases their mathematical tractability. The author, however, has amply demonstrated that the computational requirement in many cases can be reduced using simple techniques such as the *Alpha Cut* method described in this thesis and are comparable to computational requirement of other purely statistical methods. The effort in the case of Markov models, computationally, is even less when repair and failure rates are assumed as fuzzy numbers rather than random coefficients. Therefore, in Chapter VII , the author provides some concluding remarks on his attempt to develop simple Bayesian and Fuzzy-Statistical methods for treating uncertainties in the data pertaining to reliability and risk assessment studies.

In conclusion, it is hoped that, the methods presented in this thesis will give further impetus to the developement of fuzzy-statistical methods by the fusion of axiomatic structure of probability and fuzzy set theory and there by enabling us to handle problems induced by complexity of today's information based society.