Abstract

This thesis addresses developing higher order iterative methods for solving nonlinear equations in Banach spaces and their local and semilocal convergence analysis. It is one of the most important and challenging problems in scientific computing and numerical analysis. It is a difficult task in general. A number of applications can be found that give rise to these equations depending on one or more parameters. For example, the problems of Kinetic theory of gases, Elasticity and Optimization are reduced to solving nonlinear equations. It is extensively studied by many researchers, and many iterative methods of various orders are developed for their solutions. The local convergence uses information around the solution, and semilocal convergence uses information around the initial point. However, it should be noted that the convergence of iterative methods depends on an initial guess and local behavior of the function near a root. Two different types of proofs used for convergence are given. One is based on majorizing sequences, where, the sequence generated by an iterative method is majorized by the sequence generated by applying the same iterative method on a scalar function. Other is based on recurrence relations, where a problem in Banach space is reduced to a simpler problem with real sequences and functions. The major contributions of the thesis can be summarized as follows.

Higher order iterative methods are developed to approximate a root of nonlinear equations in R, and their convergence analysis are established. First, a new sixth order iterative method is developed by using the divided differences of the first order. Next, two new three-step sixth order iterative methods are developed by modifying two third order iterative methods. Newton's method is introduced as the third step in both of these methods. The derivative in the third step is approximated by the technique of linear interpolation and divided differences of the second order. The convergence analysis is provided to establish their sixth order of convergence. The efficiency indices of them are equal to 1.565. Finally, a parameter based two-step iterative method by combining two known third order iterative methods leading to a fourth order iterative method is proposed. In order to enhance the order of convergence from four to seven, its three-step extension is also developed. Here also, the increase in the number of function evaluations is reduced by approximating the involved derivative in the third step by the second order divided differences. The efficacy of these iterative methods is tested on a number of numerical examples and the results obtained are summarized in tables. The number of iterations and the total number of function evaluations used to get a simple root are taken as a performance measure. Next, the local and semilocal convergence of fifth order iterative method, a family of third and fourth order iterative methods and a family of fourth and fifth order iterative methods are established under the Lipschitz, Hölder, ω and Argyros-type conditions either on the first order or the second order Fréchet derivatives in Banach spaces. The recurrence relations are used for them. The importance of our work is that it avoids the usual practice of boundedness conditions of higher order derivatives which are a drawback for solving some practical problems. The existence and uniqueness theorems are established with error bounds for the solution. The R-order is also derived. The convergence analysis is finally worked out on different examples, and convergence balls for each of them are obtained. These examples include nonlinear Hammerstein integral equations that have many applications in chemistry and appears in problems of electro-magnetic fluid dynamics or the kinetic theory of gases, Fredholm integral equations and a boundary value problems. It also avoids the computation of derivatives of higher orders. It is found that the radius of convergence ball obtained by our approach is much larger when compared with some other existing methods. The global convergence properties of some of the iterative methods are explored by analyzing the dynamics of the corresponding operators on the complex quadratic polynomials.

Finally, conclusions and the scope of the future work are listed.

Keywords: Nonlinear equations, Local convergence, Semilocal convergence, Banach space, Fredholm integral equation, Fréchet derivative, Lipschitz condition.