Abstract of PhD Thesis

On Discretization of Sphere and Related Problems in 3D Integer Space

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The literature of digital geometry as of now contains a rich collection of work related to discretization of different geometric primitives and general surfaces in the digital or integer space. However, characterization, modeling, and algorithmic issues in connection with discrete spheres and its immediate derivatives like spherical paths and circles are not yet studied up to their merit. In this thesis, we address some of these issues and present several new theoretical findings, efficient algorithms, and computational aspects concerning the aforesaid primitive objects in the integer space. We have first shown how elementary number-theoretic analysis can lead to characterization of *naive* and *standard* models of discrete sphere, which, in turn, aids in designing efficient algorithms for their construction using simple integer operations. Based on this characterization, we work out certain effective techniques for construction of *discrete spherical geodesic paths* and circles that are taken from the intersection of a discrete sphere and a discrete geodesic plane. As a natural follow-up of this problem, we do a characterization of different topological classes arising out of the combination of discretization models of the concerned sphere and geodesic plane. We also visit the problem of construction of non-geodesic circles and present some theoretical results related to this. As a further improvement, we introduce the idea of functional gradation of *quadraginta octants* (symmetrical parts) of discrete sphere. We show how it helps in designing a fairly simple algorithm for construction of a special kind of 3D circle that maintains a better symmetry as well as bounded distance from the underlying real sphere and real plane. To address the connectivity issues of curves defined on a sphere in the discrete topological space, we propose a new model of discrete sphere called *graceful sphere*. We investigate the properties of a discrete spherical geodesic path between two voxels and show that discrete 3D circles, circular arcs, and Mobius triangles are all constructible on a graceful sphere, with guaranteed minimum thickness and desired connectivity in the topological space. Finally, to demonstrate the applicability of integer algorithms in rapid prototyping, we propose an efficient layering algorithm for construction of thick-walled discrete sphere. Experimental results have been furnished to demonstrate the usefulness and efficiency of the proposed techniques.