Chapter 1

Introduction

Determination of natural frequencies and modes of vibration of cantilever beams is of considerable importance in the design of blades used in turbomachines. It is known from investigations that failure of turbine blading is normally due to fatigue, and it occurs when vibrations take place at or near resonant conditions.

The difficulties facing the design of turbine blading are many. The magnitude of the problem lies in the fact that in some turbines there can be as many as 2000 fixed and rotating blades of different characteristics and the failure of even one of them will force a shut down. The major problem of design is a fair prediction of the natural frequencies of the turbine blading at the earlier stages by theoretical calculations. For this, the designer has to consider the effects of the following predominant factors :-

1. Taper of the blade cross-section.
2. Shear deflection and rotary inertia.
3. Rotation of the disc.
4. Stagger angle of the blade.
5. Disc radius.
6. Pre-twist of the blade.

7. Asymmetry of the cross-section of the blade.
8. Shrouding.
9. Lacing wire.
10. Root fixing.

A good deal of theoretical and experimental work has been done to enable the designer to calculate the natural frequencies, but it is still a long way to go before the last word could be said on this problem.

Since one cannot entirely depend on the theoretical calculations, it is advisable to make full scale model tests on the prototype and study the vibration characteristics, i.e., the natural frequencies and their corresponding nodal patterns. But the experimental procedure is time-consuming and very expensive to conduct tests on a number of designs. Thus it becomes essential for the designer to have theoretical knowledge as well as practical means for conducting model tests in order to predict natural frequencies at an earlier stage.

Depending on the characteristics, the turbine blading can execute either uncoupled bending or torsional vibrations or coupled bending torsion vibrations. Coupled bending torsion vibrations occur when the centre of flexure does not coincide with the centroid of the blade cross-section. When the blade is pre-twisted, vibrations are further coupled between the two bending modes. The differential equations of motion for the case of a pre-twisted turbine blade with asymmetric cross-section mounted on a rotating disc with a stagger angle are very complex, and complete solution of such equations is difficult to obtain. Also, the natural frequencies determined are affected by shear deflections, rotary inertia, fibre bending, root fixing and Coriolis accelerations. When the blades are shrouded, the coupling between the blade and the shroud is to be further taken into account. Thus, theoretically it is an uphill task to determine the natural frequencies of an actual turbine blade with all the effects mentioned above.

Various investigators in the field of turbine blade vibrations have solved the differential equations of motion by taking into account the individual aspects mentioned above, and determined their effects on vibrational characteristics.

This thesis is aimed to present the theoretical solutions of differential equations of motion of turbine blades with the effects of taper, shear deflections and rotary inertia, disc rotation, stagger angle of the blade, disc radius, pre-twist of the blade and the asymmetry of cross-section. Methods of Ritz, Galerkin and Collocation are used to determine the individual effects of parameters just mentioned and the results obtained are compared with experimental values of the author or others as given in chapters 4, 5, 8, 9, 10, 12, 13, 18 to 23, 26 and 27, and a favourable agreement between them has been achieved. Numerical procedures based on polynomial expressions for deflections are also developed to determine the frequency equation of a discrete system and their solutions presented for the cases of uncoupled bending and torsion vibrations of stationary and rotating cantilever blades and coupled bending torsion vibrations of stationary cantilever blades. The thesis consists of the following chapters :-

1. Introduction.
2. Literature Survey.
3. Fundamental flexural vibration of a tapered cantilever beam treated by Galerkin method. - The derivation of an equation to determine the fundamental flexural frequency of a tapered cantilever beam with rectangular cross-section is presented in this chapter.
4. Effect of taper on lateral vibrations of cantilever beams treated by Galerkin method. - In this chapter, the results obtained for the first three modes of flexural vibration by digital computer are compared with experimental results.
5. Effect of taper on lateral vibrations of cantilever beams treated by Collocation method. - Collocation method is used to determine the first three modes of flexural vibration of tapered cantilever beams and the results obtained are compared with experimental values.
6. A numerical process for the determination of uncoupled lateral frequencies of a cantilever beam. - Using Myklestad equations a numerical process is developed to determine the frequency equation of a discrete mass system. An example is considered and the results obtained are compared with theoretical results of Myklestad.

7. Fundamental torsional vibration of a tapered cantilever beam with rectangular cross-section treated by Collocation method. - Collocation method is applied to derive an equation for the fundamental torsional mode of a tapered cantilever beam with rectangular cross-section.

8. Torsional vibration of tapered cantilever beams treated by Collocation method. - This chapter deals with the first two modes of tapered cantilever beams and the results obtained by the Collocation method on a digital computer are compared with experimental values.

9. Torsional vibration of cantilever beams with thickness taper treated by Galerkin method. - The effect of depth taper is considered by Galerkin method and the theoretical values obtained for the first two modes of torsional vibration are compared with experimental values.

10. A numerical procedure for determining natural frequencies in torsional vibration of cantilever beams. Using Holzer's equations a numerical procedure is developed to determine the frequency equation of a discrete inertia system and the results obtained for a practical case are compared with experimental values.

11. Correction factors for the effect of taper on the torsional vibrations of cantilever beams determined by Holzer method. - Using Holzer's method and one hundred stations along the length of the beam, the results obtained with the aid of a digital computer for the effect of taper are plotted for the first three modes.

12. Experimental determination of uncoupled natural frequencies and comparison with theoretical values obtained by Correction factors. - Experimental process for determining the natural frequencies is described and the results obtained are compared with theoretical values.

13. Effects of shear deflection and rotary inertia on tapered cantilever beams treated by Ritz method. - The equation of motion is solved by Ritz energy process and the effect of taper on first three flexural modes of vibration is studied. The theoretical results are compared with experimental values.

14. Lateral vibration of rotating cantilever beams treated by Galerkin method. - The equation of motion is solved by Galerkin method and the results obtained are reported in graphical form.

15. A numerical process for flexural vibrations of rotating cantilever beams. - Using Myklestad equations, a numerical procedure is developed to determine the frequency equation of a rotating cantilever beam and the results obtained are compared with the Myklestad method.

16. Non-linear vibration of rotating cantilever beams treated by Ritz method. - Using the Ritz energy process, the non-linear partial differential equation is solved to determine the response of a rotating cantilever beam.

17. Non-linear vibration of rotating cantilever beams treated by Ritz averaging process. - The non-linear partial differential equation is solved by Galerkin method taking into account the effect of damping, to determine the response of

a rotating cantilever beam.

18. Effect of pre-twist on flexural vibrations of cantilever beams treated by Galerkin method. - The two coupled differential equations are solved by Galerkin method and the theoretical results obtained are compared with experimental values.

19. Effect of pre-twist on torsional vibration of cantilever beams treated by Collocation method. - Collocation method is used, to solve the differential equation of motion and the results obtained are compared with experimental values.

20. Torsional vibration of pre-twisted beams including the effects of fibre bending, treated by Galerkin method. - The differential equation of motion is solved taking into account the effects of fibre bending, and the results obtained are compared with experimental values.

21. Flexural vibration of pre-twisted tapered cantilever beams treated by Galerkin method. - The two coupled differential equations of motion are solved by Galerkin method and the results obtained are compared with experimental values.

22. Torsional vibration of pre-twisted tapered cantilever beams treated by Collocation method. - In this chapter the Collocation method is used to solve the differential equation of motion and the results obtained are checked with experimental values.

23. Torsional vibration of pre-twisted tapered cantilever beams treated by Galerkin method. - Galerkin method is used to solve the differential equation of motion and the results obtained are compared with experimental values.

24. Effects of pre-twist and rotation treated by Extended Holzer method. - In this chapter the effects of pre-twist and rotation on flexural vibration of cantilever beams are studied by using an extension of Holzer method.

25. A numerical process for coupled bending torsion vibrations of cantilever beams. - Using Holzer and Myklestad equations, a numerical process is developed to determine frequency equation of a discrete mass system executing coupled bending torsion vibrations.

26. Coupled bending bending torsion vibrations of straight uniform cantilever beams treated by Galerkin method. - The three coupled differential equations are solved by Galerkin method and the theoretical results obtained are compared with those of other authors and experimental values.

27. Coupled bending bending torsion vibrations of cantilever beams treated by Collocation method. - Collocation method is used to solve the coupled differential equations and the results obtained are compared with Galerkin's process and experimental values.

28. Conclusions. - The method of Galerkin, Collocation, Ritz and the numerical methods developed in this work are briefly commented on their relative merits and demerits in connection with the turbine blade problems. A brief summary of the conclusions on the individual chapters is included.

29. Scope of future work. - The scope of future work in turbine blade vibration is briefly indicated.

30. References.