SYNOPSIS

Ground water is the most widely distributed natural resource and the demand for its exploitation has been rising rapidly in recent times. Wells are used for the recovery of ground water for water supply and also for the purpose of drainage. For the design of either system of wells, a knowledge about the relation between the discharge of the well and the drawdown is necessary.

The aquifers from which water is withdrawn may be either confined or unconfined.An unconfined aquifer is a permeable bed overlying a relatively impervious layer. Its upper boundary is formed by a free water table or phreatic level under atmospheric pressure. A confined aquifer is a permeable bed whose upper and lower boundaries are impervious layers. The wells may penetrate the aquifer either fully or partially. A fully penetrating well is adequately perforated throughout the saturated thickness of the aquifer. If the pumping well is not fully penetrating, it is called a partially penetrating well. In the case of a partially penetrating well, water enters from the sides as well as from the bottom, while in the case of a fully penetrating well, water enters only from the sides. Sometimes, the partially penetrating well may

prove more economical as the cost of the well may be less for no significant reduction in the discharge.

The hydraulics of wells in confined aquifers have been studied rather exhaustively in the literature. But the studies on wells in unconfined aquifers have been rather limited. The flow towards wells in unconfined aquifers constitutes a difficult class of problems due to the fact that one of the boundaries of the seepage domain, namely the free surface, is initially unknown and is sought to be obtained by a series of trials in the course of the solution. While the differential equation for the flow remains linear as in the case of confined seepage, the boundary conditions on the free surface are non-linear. There exists to-date no rigorous mathematical technique for this type of axisymmetric unconfined flow, although mathematical solutions do exist for plane unconfined flow, in simple situations, by the application of the complex variable techniques. For example, if there is only a free surface or if there is only a surface of seepage in a two-dimensional problem, it is possible to solve the problem by conformal transformation, using hodograph plane or Zhukovsky functions. But, when there is combination of both the free surface and the surface of seepage in a two-dimensional problem, the techniques of complex variable fail for unconfined seepage. It is well known that the complex variable techniques cannot be extended to axisymmetric seepage problems.

However, Dupuit solved the problem of seepage to a fully penetrating well in an unconfined aquifer introducing some well known approximations. It is known that the Dupuit's expression for discharge is exact, although the free surface predicted by that expression is incorrect in the vicinity of the well. For a partially penetrating well in an unconfined aquifer, empirical equations for the discharge have been presented by Forchheimer and Kozeny. Both of these equations are known to be largely in error. Boreli presented a formula for the discharge and free surface around a partially penetrating well based on the solutions of only eight cases, using the relaxation method. In view of the limited number of cases studied, the results of Boreli are applicable over a narrow range and are of limited accuracy. Further, they are not applicable in the range of shallow penetration of wells.

It is thus noted that the studies made till now on the flow towards partially penetrating wells in unconfined aquifers have been extremely limited. Hence in the present investigation, a comprehensive study of the flow conditions around such wells including fully penetrating wells is taken up covering a wide range of the geometrical parameters defining the seepage domain. For this study, the aquifer is taken to be homogeneous and isotropic and the flow is considered to be in steady state. The main study relates to the flow conditions governed by Darcy's law. A few cases of flow towards fully penetrating wells, governed by Forchheimer's non-linear seepage law, is also included.

The solutions for the axisymmetric unconfined flow problems can be obtained experimentally, using sand models or electrical analog models and numerically by employing the finite difference method or the finite element method. In sand models, the determination of free surface is difficult because of capillarity. This makes difficult the direct observation of the elevation of water surface in sand grains adjacent to the well with rapidly fluctuating drawdown. However, it is possible to locate the free water surface by the electrical analogy method. The difficulty in such models is that the free surface must be carved to a radial profile located by trial and error which involves a tedious process. Hence a numerical analysis of the seepage field, using a digital computer, is taken up in the present investigation.For this purpose, the powerful technique of the finite element analysis, based on the variational formulation of the problem, is adopted in view of the several advantages offered by this technique over the finite difference method particularly in connection with the direct determination of discharge.

In the finite element analysis, the seepage domain around the partially penetrating well in an unconfined aquifer has been discretized as an assemblage of network of triangular elements having three nodes. Variation of the hydraulic potential in an element has been assumed to be linear.Equations relating the hydraulic potential and the flow rate at each node of an individual element are developed. The equations of the individual elements are combined to yield the equations corresponding to the entire flow domain. Considering the aspects of convergence of the values of the discharge and also the convergence of the free surface ordinates close to the well for a particular problem, it has been proposed to keep the number of elements approximately at 450 and the number of nodal points approximately at 250 in the finite element models, with the actual number varying from problem to problem.A scheme of partitioning is adopted, as a result of which the requirements of the computer storage are cut down, making it possible to solve the problems involving a large number of unknowns (hydraulic potential at nodes) on small and medium size computers.

Three iterative approaches have been examined for the delineation of the free surface around the well and the best iterative approach has been employed for the solution of the problem.

Computer programs, coded in Fortran IV, have been developed for the solution of the seepage problem. These programs are versatile and can handle a wide variety of seepage flow conditions, namely plane two-dimensional flows or axisymmetric flows; confined flows or unconfined flows; and Darcy flows or non-Darcy flows (based on Forchheimer's law). The necessary input data for the problem of unconfined seepage towards wells consists of the radius of the well, the height of the well bottom above the impervious base, the depth of water in the well, the depth of water at the inflow boundary and the radius to the inflow boundary. The necessary information for the discretization of the seepage field, like the nodal numbers of the elements, the coordinates of the nodes, the nodal numbers of the first node and last node in each partition, the number of the first and last elements in each partition, the nodal number of the boundary nodes and the type of boundary conditions on them, etc. are all generated automatically in the computer. This gives an advantage in handling the minimum data for input. This automatic generation of the mesh provides for a closer spacing of nodes near the well for improved accuracy.

The validity of the computer models of the finite element analysis has been tested by solving some problems with known solutions.

The existing formulae for unconfined seepage towards well, like those of Dupuit, Forchheimer, Kozeny, Babbitt and Caldwell and Hall, require the coordinates of a point on the free surface, located far away from the well. The radius to such a point is considered as the radius of influence of the well. However, these formulae do not indicate a precise criterion to choose that point. Hence, the results of these formulae, particularly for the free surface, vary with the location of the point. So in the present investigation, a point on the free surface, far away from the well, has been chosen as a standard reference point for the development of the formulae for the discharge and the free surface around the well. The radial distance of this reference point has been taken equal to twice the height of the free surface at that point, measured above the impervious base. This reference point is located in a domain where the free surface is closely represented by Dupuit's formula. However, the results of the present study can be extended to problems, with a different reference point.

The radial coordinates of the nodes on the free surface have been fixed at some selected percentage of the distance between the well face and the inflow boundary in the finite element models to facilitate the development of the equation of the free surface around the well.

Thirtyfive cases of partially penetrating wells and fifteen cases of fully penetrating wells covering a wide range of geometrical parameters defining the seepage domain have been solved on a digital computer. For purposes of the analysis of results, the fully penetrating wells have been considered as particular cases of partially penetrating wells.

Equations for calculating the discharge and free surface profile around partially penetrating wells are obtained, separately for the ranges of shallow and deep penetration. The results obtained are compared with those of the existing formulae, where available.The zones of validity of the various formulae are identified. The formulae, developed in the present investigation, are valid over a wide range and are shown to entail negligible errors in comparison to those of the other formulae.

It is known that in some coarse grained soils, Darcy's law is not applicable near the well due to high velocity of flow there.In the present investigation, a few cases of non-Darcy free surface flow towards wells governed by Forchheimer's non-linear seepage law have also been studied, using the finite element models. These studies have been restricted to the fully penetrating wells only. Two types of problems have been studied. In one type, the entire seepage domain around the well has been assumed to experience non-Darcy flow. In the other type, the flow domain closer to the well has been assumed to have non-Darcy flow with the rest of the seepage domain having Darcy flow.

An equation for the discharge into a fully penetrating well, governed by Forchheimer's law has been deduced, using Dupuit's assumptions.

A qualitative appraisal has been made of the difference between Darcy and non-Darcy flow conditions in relation to the discharge into the well, the form of the free surface curve and the potential gradients close to the well. It is found that non-Darcy seepage flow towards wells entails lower discharge, higher free surface heights and higher potential gradients close to the well in comparison to Darcy flow conditions. Further, it is shown that it would be adequate to consider a small region around the well to be affected by non-Darcy flow for the purpose of analysis.

The principal contribution, made in the present investigation, has been to develop the equations for the discharge and for the form of the free surface around partially penetrating wells and fully penetrating wells in unconfined aquifers. The formulae have been separately established for the ranges of shallow and deep penetration.

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