INTRODUCTION

1. Importance of plate theory

During recent years, the importance of the theory of bending of plates has gradually increased in engineering and technology. In engineering instrumentation, structural analysis or designing, problems often arise which involve plate structures. For instance, a variety of plates concerning the bending of circular plates occur in the design of boilers, locomotive engines and steam turbines. Circular ring plates have found some engineering applications as diaphragms and high pressure expansion members and plate valves in compressers. Other notable examples are steel plates of ship hulls submitted to the action of hydrostatic pressure, concrete and reinforced concrete slabs under the action of lateral loading and thin-walled tanks and containers of various shapes submitted to the action of internal or external pressures. Particularly, at the present time, thin-walled structures find a wide application in the modern development of space technology. The great variety of the construction elements mentioned above shows from the practical stand point, the particular importance of the plate problems.

2. Basic Equations for the bending of thin plates (Classical small deflection theory)

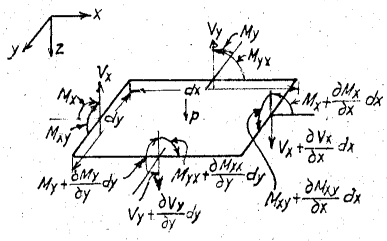
The Kirchoff's classical theory of thin plates (1850) bent with small deflection, that is, when the deflection of the middle plane is small compared with the thickness which is also supposed small with respect to other dimensions of the plate, has been formulated under the following assumptions:

i) The normals of the middle plane before bending are deformed into the normals of the middle plane after bending.

ii) The stress perpendicular to the plane of the plate is small compared with the other stress components and may be neglected in the stress-strain relations.

iii) The middle plane remains unstrained after bending.

We choose the coordinate axes so that the x- and y- axes are in the middle plane of the plate and the z-axis is perpendicular to the middle plane. Consider a section of the plate parallel to xz plane. After bending, a point A on the middle plane is deflected to A' with a deflection w. Since the deflection is small, the displacements u and v in x and y direction will respectively be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$M_{yx}+\frac{\partial M_{yx}}{\partial y}dy$
$V_y+\frac{\partial V_y}{\partial y}dy$
$M_{xy}+\frac{\partial M_{xy}}{\partial x}dx$

FIG. 0.1

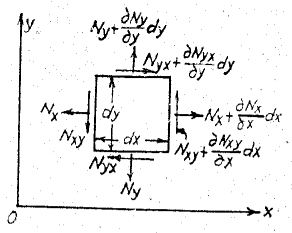
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FIG. 0.2

$$u = -z\frac{\partial w}{\partial x}, \quad v = -z\frac{\partial w}{\partial y} \tag{0.2.1}$$

Hence, the strain components are

$$\epsilon_x = -z\frac{\partial^2 w}{\partial x^2}, \quad \epsilon_y = -z\frac{\partial^2 w}{\partial y^2}$$
$$\gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y}. \tag{0.2.2}$$

The stresses derived from the above strain components are easily seen to be

$$\sigma_x = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$
$$\sigma_y = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \tag{0.2.3}$$

and $$\tau_{xy} = -\frac{Ez}{1+\nu}\frac{\partial^2 w}{\partial x \partial y}$$

With these relations, the bending and twisting moments per unit length acting on any section of the plate parallel to the xz and yz planes (Fig.0.1) can be obtained by integration. Thus,

$$M_x = \int_{-h/2}^{h/2} \sigma_x z\, dz$$

Since w does not depend upon z, we have

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \tag{0.2.4}$$