ABSTRACT

Directional data cannot be treated in the same way as Euclidean data due to disparate topologies between Euclidean and other spaces. This thesis addresses the estimation for parameters of directional distributions.

The two-parameter Langevin distribution has been widely used for analyzing directions. Chapter 3 considers the estimation of Langevin mean direction in its Cartesian and angular forms. Bayes estimators for the mean direction are derived with respect to several priors. The equivariant point estimation is introduced under different transformation groups. The maximum likelihood estimator (MLE) is shown to satisfy many decision theoretic properties such as admissibility, minimaxity, the best equivariance and risk-unbiasedness under various loss functions. These results extend and unify earlier results on the optimality of the MLE. These findings are also established for the problem of simultaneous estimation of mean directions of several independent Langevin populations. Further, estimation of a common mean direction of several independent Langevin populations is studied. Simulation studies are conducted to analyze and compare risk functions of the MLE and Bayes estimators.

Chapter 4 deals with M-estimators, restricted M-estimators and R-estimators for the mean direction of a rotationally symmetric distribution on the unit hypersphere. The influence function and asymptotic distribution of an R-estimator are derived for a general density. The spherical median is shown to dominate over various estimators in terms of gross error sensitivity (GES). Asymptotically most efficient estimators are obtained in classes of restricted M-estimators and R-estimators. We present explicit expressions of asymptotic distributions and influence functions of various estimators for the mean directions of Langevin and mixture Langevin models. On the basis of these expressions, their asymptotic relative efficiencies (AREs) and GESs are compared. As a consequence the trade-off between robustness and efficiency amongst various estimators has been explored.

In decision theoretic estimation of parameters in Euclidean space $\mathbb{R}^p$, the action space is chosen to be the convex closure of the estimand space. In Chapter 5, the concept has been extended to the estimation of circular parameters of directional distributions. As these distributions are of curved nature, existing methods for distributions on $\mathbb{R}^p$ are not immediately applicable here. Since circle is the simplest one-dimensional Riemannian manifold, we employ various concepts of manifolds to develop sufficient conditions for the inadmissibility of estimators for circular parameters. Further invariance under a compact group of transformations is introduced in the estimation problem and a complete class theorem for equivariant estimators is derived. This extends the results of Moors (1981) on $\mathbb{R}^p$ to circles. The findings are of special interest to the case when circular parameter is truncated. The results are implemented to a wide range of directional distributions to obtain improved estimators of circular parameters.

Chapter 6 focuses on the estimation of the concentration parameter κ of a Langevin distribution. Since the normalizing constant in this distribution contains the modified Bessel function of the first kind with the argument κ, estimators for κ using standard procedures cannot be obtained in closed or explicit form. We derive analytic expressions up to third order for the bias, mean squared error (MSE) and variance of the MLE of κ when Langevin mean direction $\boldsymbol{\mu}$ is known. These expressions are derived up to first order when $\boldsymbol{\mu}$ is unknown. Using these expressions in selective manner and applying various bias reduction techniques, several nearly-unbiased estimators for κ are proposed. A massive simulation study is carried out to compare the biases and MSEs of these new and earlier proposed estimators. Many of our proposed estimators are seen to offer substantial improvements over the earlier ones in terms of bias and MSE. Specific recommendations about the usage of these estimators are made based on our study.

Keywords: Admissibility, asymptotic relative efficiency, convexity, directional data, equivariant estimator, gross error sensitivity, M-estimator, minimaxity, orthogonal group, projection, R-estimator, restricted estimation problem, techniques for bias reduction.