Abstract

The present thesis deals with the operator ideals of bounded linear operators and the sequence spaces obtained with the help of the generalized means and difference operators.

Some classes of operator ideals by using s-number of bounded linear operators and infinite matrices are introduced. It is shown that the operator ideals are quasi-Banach ideals with suitable quasi-norms. Some inclusion relations among the operator ideals which are generated by various s-numbers, e.g., approximation number, Gelfand number, Kolmogorov number are established. Duals of the operator ideal are also obtained. Further, a vector-valued sequence space by using a sequence of Orlicz functions and an infinite matrix is introduced and studied. This sequence space includes many well known sequence spaces such as Musielak-Orlicz, Cesàro-Orlicz, Orlicz-Lorentz sequence space as particular case. It is also shown that the vector-valued sequence space is a Banach lattice. Some properties of Banach lattice, namely, uniform monotonicity, strict monotonicity are also discussed.

New difference sequence spaces defined by the generalized means are introduced and studied in detail. Using higher order difference operators and generalized means, many of the important classes of sequence spaces are generalized. Some topological and geometric properties of these sequence spaces are studied and the α-, β-, γ- duals of these sequence spaces are obtained. Necessary and sufficient conditions for the matrix transformations between these sequence spaces and the classical sequence spaces such as c_0, c, l_∞ are derived. The characterization is obtained for the matrix maps between the sequence spaces, to be compact operators by using the Hausdorff measure of non-compactness.

Keywords and Phrases: Bounded linear operator, Compact operator, Operator ideal, s-number, Approximation number, Orlicz function, Musielak-Orlicz function, Uniform Opial property, Difference operator, Generalized means, α-, β-, γ- duals, Matrix transformations, Hausdorff measure of non-compactness.