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PhD Thesis Title: On Farey Sequence and Farey Table with Digital-geometric Applications to Image Analysis

Abstract

Farey sequences, introduced by the renowned mathematicians like John Farey, Charles Haros, and Augustin-Louis Cauchy over 200 years ago, is quite well-known by today in *theory of fractions*, but its computational perspectives are possibly not yet explored up to its merit. In this thesis, we present some new theoretical findings, efficient algorithms, and computational aspects related with a Farey sequence along with its digital-geometric applications to image analysis. From an original Farey sequence that contains only simple and proper fractions in ascending order, we obtain an *augmented Farey sequence* (AFS) for its applicability to 2D image analysis. The *ranks* of the AFS fractions are stored in an *augmented Farey table* (AFT), proposed by us, which provides an efficient solution to the *rank problem*, thereby aiding in and speeding up different digital-geometric techniques. As the size of an AFT increases quadratically with the order of the Farey sequence, we have also designed an efficient algorithm for finding the *closest rank* of any fraction in an AFT of a lower order. For the reverse problem on *order statistics*, an AFT is not efficient for finding the fraction of a given rank. Hence, as an improvement, we have proposed the *rank-based Farey table* (RFT), which not only requires lesser storage space but also solves both the rank and the order statistics problems quite efficiently. For space reduction and with a focus on the rank problem only, we have also proposed two (lossy) compression schemes to obtain a *compressed Farey table* (CFT) from an AFT. Necessary analysis has been done to derive the error bound in a CFT. The idea of CFT comes up with the notion of *approximate rank*, which could be useful in practical scenarios where *maximum errors* are pre-specified. To demonstrate the applicability of AFT, two well-known image-analytic problems have been revisited, one involving *polygonization* in digital images and the other related to *skew correction* in digitized documents. Experimental results have been furnished to demonstrate the usefulness, efficiency, and robustness of the proposed techniques based on AFT.