Chapter 1

Introduction and Literature Review

1.1 Motivation

In recent years, the dynamic behavior of thin isotropic plates has gained considerable importance due to its widespread technical usage ranging from automotive, shipbuilding, aerospace to civil industries. The analyses of these type of structures are rapidly increasing for noise control, vibration control or for the study of its dynamic stability. Again, these structures are subjected to unwanted instances of high vibration which can generate local damages in the structure. These flaws are extremely dangerous and can affect the stability of the structure. Therefore, early detection and localization of these damages are very important to the safety of the structure.

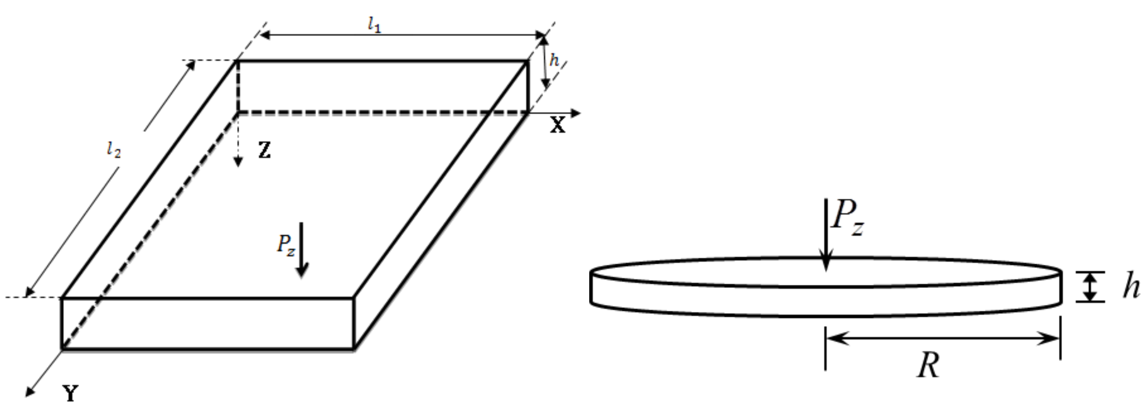
Several condition monitoring techniques are available in the literature, out of which vibration and acoustic signal based monitoring are the most suitable for global monitoring of a structure. Vibration based crack detection methods are very accurate, but their performance heavily relies on the position of the sensors and also suitable signal processing algorithms. Several models were developed in the past for accurate crack detection and prognosis. These linear models are first validated and then updated using an experiment. After that, these models can be used to detect a crack with sufficient accuracy and are also helpful for future prediction. So, the major advantage is that once validated this method replaces the costly experiment. These linear models are found to be effective in monitoring of civil structures, sea platforms and numerous industrial machineries. However, it may not be possible to mount a sensor directly to some structure in certain circumstances such as elevated temperature, hazardous environment, etc. In this context, acoustic signal monitoring is found to be the best way as it can be monitored in a non-contact way. Structural vibration generates sound radiation into the surrounding fields which is detectable by an array of microphones. So, if the sound field is known the vibration pattern of the structure can be found by inverse methods and then the regular vibration based techniques can be used for condition monitoring purpose. Some acoustic quantities as radiation efficiency, sound power can also be studied for the same purpose. Therefore, acoustic signal based crack detection methods are an emerging field of study.

Sometime the linear model based crack detection become erroneous, especially when considerable nonlinearities are present in the structure as the large deflection of an aircraft wing. These nonlinearities make the crack detection process difficult as the model or signal processing techniques used in the process are linear. The presence of a crack also develops nonlinearity in the structure which is influenced by the presence of other nonlinearities. The effect of these nonlinearities is prominent only in some nonlinear phenomena such as jump, period doubling, bifurcation and chaos. So, accurate crack detection is only possible in the nonlinear domain and for this reason the development of different nonlinear models for crack detection and prognosis is growing importance.

1.2 Literature Review

In this section, a survey of available literature in the area of vibration of cracked plates is performed. At first, some basic introduction to plate is presented. Then the articles concerning linear and nonlinear plate theory are documented. The sound radiation from plate structures are reviewed next. Then different damage identification techniques used in the prior literature are discussed. Next, modeling aspect of damaged plate structures is reviewed from previous literatures.

Plates are widely used in structural components as in buildings, bridges, helicopter rotor blades, ships, sea platforms, flexible satellite manipulators, satellites, aircraft wings and subsystems of more complex structures. Therefore, they carry a great practical interest to civil, mechanical, marine and aerospace engineers. They can be of various shapes as rectangular, circular, triangular, etc. and can be subjected to various types of boundary conditions. Use of different materials (it may be a homogeneous, composite, sandwich, viscoelastic or a functionally graded material) also changes the analysis type. Analyses can be performed in static or dynamic domain. In static domain, the plate theories are broadly classified as a thin plate theory and thick plate theory. The thin plate theory is applicable when the thickness to width ratio is less than 0.1. Two widely used plate theories in engineering problems are Kirchhoff's theory of thin plates and Mindlin-Reissner plate theory of thick plates. In this research, thin plate theories applicable for homogeneous materials are mainly reviewed and discussed.



(a) (b)

Fig. 1.1 (a) Rectangular plate and (b) Circular plate subjected to forced vibration.

Thin plate theories can also be categorised on the amount of transverse deflection in comparison with the other plate dimensions. So, they are typically divided as

(a) *Small deflection theory* (linear) is used for deflection less than twenty percent of the thickness. This is the most common theory and it is applicable for nearly all the civil structures, machineries, sea platforms.

(b) *Moderately large deflection theory* (Nonlinear) is used when the deflection is in multiple of the plate thickness but much less than the plate side length. This type of deformation occurs for an aircraft wing. Perturbation techniques are used to solve this type of problem.

(c) *Very large deflection theory* (Highly nonlinear) is applicable when the transverse deflection is close to the plate side length. This type of deformation occurs when a structure is subjected to a shock load. Perturbation techniques cannot solve this type of problem and usually some approximate techniques are used.

So, depending on the classification of the plate deflection the solution can be relatively simple or highly complex, and sometimes impossible without the implementation of approximating techniques. In this study, only linear and moderately nonlinear plate theories are discussed.

1.2.1 Linear theory

Investigation on the plate theory has begun since the end of the 19th century with the work of German physicist Chladni who discovered various modes of free vibrations experimentally in 1827. With the advent of time, it has developed into a field of great practical interest dealing with increasingly complicated problems. The first mathematical solutions to the free vibration problem of the membrane theory of plates were formulated by Euler, in 1766, and his student Bernoulli, in 1789. Lagrange, in 1813, developed the

the first correct governing equation for the free vibration of plates. The first correct differential equation of plates subjected to distributed static lateral loads was developed by Navier in 1836. Later, the same differential equation was derived by Kirchhoff, in 1887, from a different energy approach (which is widely used now). The governing differential equation for a rectangular plate (Fig. 1.1a) is as follows (Ventsel and Krauthammer, 2001):

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=-\rho h\frac{\partial^2 w}{\partial t^2}+P_z \tag{1.1}$$

where D is the flexural rigidity of the plate, ρ is the density, h is the thickness, P_z is the force and w is the transverse displacement. Similarly, the governing equation for circular plate (Fig. 1.1b) in polar coordinates can be expressed as (Ventsel and Krauthammer, 2001)

$$D\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)+\rho h\frac{\partial^2 w}{\partial t^2}=P_z \tag{1.2}$$

The first correct governing equation of plate vibrations is attributed to Sophie German in the early 19th century (Rayleigh, 1945). Rayleigh (1945) proposed several methods to determine the natural frequencies in his famous book *Theory of Sound* which was first published in 1877. Tomotica (1936) presented work on the vibrations of square plates. Young (1950) used the Ritz method to compute the approximate natural frequencies of clamped square plates. The first comprehensive collection of solutions for rectangular plates was proposed by Warburton (1954), using beam functions. The main advantage of using beam functions is that different boundary conditions can be handled with relative ease. The frequency equation was fully developed after seven years

(Mindlin, 1960). To extend the range of applicability of the classical theories to higher frequencies, the refined dynamic theories of beams were introduced by Timoshenko (1921) which include the effects of transverse shear and rotary inertia. Later, Mindlin (1951) generalized the Timoshenko beam theory to the plate and also compared it with the exact dynamic elasticity theory of Lord Rayleigh, by means of which a shear factor term was introduced.

1.2.1.1 Vibration of thin plate

In this section, small amplitude vibration of a thin rectangular plate is reviewed and discussed. Small deflection of a vibrating thin plate has been widely studied and a variety of literature is available concerning such linear vibration problems. Leissa (1973) presented a review paper on free vibrations of rectangular plates, and comparisons were made with Warburton's useful approximate formulas. Later, the author published a monograph on the vibration of plates (Leissa, 1993), in which he reviewed the contemporary literature on plate vibrations. One of the classical references for plate problems is *Theory of Plates and Shells* by Timoshenko (1940). Laura and Duran (1975) used a simple polynomial approximation and the Galerkin's method to determine the response of a simply supported and clamped rectangular plate subjected to a harmonic excitation. The agreement of approximate solution with the exact solution was found to be better in case of clamped boundary conditions than the simply supported case. It was also shown that one term approximation yields good results for simply supported plate. Vijayakumar and Ramaiah (1978) investigated the natural frequencies of a clamped rectangular plate using the Rayleigh-Ritz and modified Bolotin method. The results were observed to be accurate for higher modes when |m-n| is large, where m and n are the mode numbers. Bhat (1984) first introduced the orthogonal polynomials to find the natural frequencies and mode shapes for different plate boundary conditions. These polynomials are generated by Gram-Schimdt process. However, ill-conditioned matrices result when higher order polynomials are used. So, this method is particularly suitable for eigen-analysis of lower modes. The axisymmetric vibration of circular and its analog in elliptical plates was studied by Rajalingham and Bhat (1993) using some boundary characteristic orthogonal polynomials in the Rayleigh-Ritz method. Simply supported, clamped and free boundary conditions were studied and first six natural frequencies were reported. However, the author commented that the method may not be applicable when the major axis of ellipse is very large compared to its minor axis.

1.2.2 Nonlinearities

Nonlinearities exist in an equation of motion when some product of variables, or their derivatives, exists. Nonlinear problems are of interest to the scientific and engineering communities because most physical systems, such as structures, behave as weakly nonlinear or quasi-linear systems. These systems generally exhibit similar behavior to that of their linear counterparts. Additionally, they also show some phenomena which do not exist in the linear domain. Therefore, for weakly nonlinear systems, the usual starting point is still the identification of natural frequencies and mode shapes of the system. Then, the nonlinear natural frequency and mode shapes can be found by different perturbation techniques. A nonlinear system also exhibits interesting phenomena such as jumps, saturations, sub-harmonics, super-harmonics, combination resonances, self-excited oscillations, parametric excitations, bifurcations and chaos which cannot be found in a linear system. Detailed explanations about the various types of nonlinearities, with examples, can be found in the books of Nayfeh and Mook (1979). The sources of nonlinear behavior can be classified into following categories

1. *Geometric nonlinearity* is important for systems with large deflections, or systems that may fail due to buckling. When plates are deflected beyond its thickness, linear theory loses its validity and produces incorrect results. The geometric nonlinearity may appear in plates due to two reasons, namely the nonlinear strain-displacement relationship, and the nonlinearity due to the coupling of in plane and transverse displacement fields. Mid-plane stretching of the plate may occur due to the second reason.
2. *Material nonlinearity* appears when the stress-strain relationship of the material become nonlinear i.e., when the Hooke's law becomes invalid. Typical examples are rubber and the behavior of ductile material after the yield point.
3. *Boundary condition nonlinearity* caused by the nonlinear boundary conditions. Examples of such phenomena include the use of a nonlinear spring or damper on the edge of a plate, or the case of a nonlinear spring in a mass-spring-damper system.
4. *Inertia nonlinearity* generates from the velocities or accelerations in the equation of motion. The kinetic energy of the system is the source of inertia nonlinearities. Examples include centripetal and Coriolis acceleration terms.
5. Besides these categories, impacts, backlash, fluid effects and damping are also capable of categorizing other types of nonlinearities which exist in structures (Malatkar, 2003).

In this study, geometric nonlinearities are mainly reviewed and discussed. In linear systems, the strain-displacement relationship is linear, but when the mid-plane stretching occurs due to the large deflection of a plate this relation is no longer linear.

1.2.2.1 Nonlinear plate theory

A selective literature survey on moderately large deflection of a plate is discussed here. The discovery of nonlinear theory which accounts for bending and stretching of a plate is credited to Kirchhoff (1824-1887). Similar problems are discussed in the book of Nayfeh and Mook (1979). Another interesting book in this area has been published by Chia (1980). The book *Nonlinear Analysis of Plates* covers a wide range of problems concerning different nonlinear plates and also the methods to approach these problems.

For moderately large deflection of the plate when the governing equations are written in terms of deflection, it results in fourth order nonlinear partial differential equations (von Kármán, 1910). The governing differential equations of a rectangular plate are (Ventsel and Krauthammer, 2001) written as follows:

$$\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}=Eh\left[\left(\frac{\partial w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{1.3a}$$

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=P_z+\frac{\partial^2\Phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2\Phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\frac{\partial^2\Phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y} \tag{1.3b}$$

where Φ is the stress function. Equations are coupled, nonlinear and their solutions are very difficult to obtain. The first equation is an equilibrium equation and the second one is a compatibility equation. Several authors have tried to solve these equations approximately using a perturbation procedure (Chu and Hermann, 1956), asymptotic expansion (Yamaki, 1961), finite difference methods (Bayles et al., 1972), finite element methods (Mei, 1969) and analogue computer methods (Ramachandran and Reddy, 1972). Levy (1942) substituted a double Fourier series solution into the equations for rectangular plates and evaluated the coefficients. Berger (1955) first decoupled the nonlinear equations of von Kármán (1910) by neglecting the strain energy due to the second invariant of middle surface strains. Though the solutions are strictly limited to few boundary conditions and also applicable for symmetric loadings only, it is very useful for engineering calculations. Several authors have used the Berger's equations because of its simplicity. Chu and Hermann (1956) investigated the nonlinear vibration of rectangular plates using the coupled equations of von Kármán. Simply supported and hinged boundary conditions were used in that investigation. The general solution of these equations is unknown but the first approximations were obtained by these authors using a perturbation method and the principle of conservation of energy. Nash and Modeer (1959) followed the Berger's hypothesis and developed the dynamic analog of Berger's equations as follows:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=-\rho h\frac{\partial^2 w}{\partial t^2}+P_z+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2} +2N_{xy}\frac{\partial^2 w}{\partial x \partial y} \tag{1.4}$$

where the last three terms are the coupling between the membrane and bending terms. N_x, N_y and N_{xy} are the membrane forces which can be derived from the strain-displacement relations. Similarly, the governing equation of a circular plate can also be derived. The authors (Nash and Modeer, 1959) showed that the results were in excellent agreement with that of Chu and Hermann (1956) for simply supported plates. Yamaki (1961) investigated the large amplitude vibration of rectangular and circular plate using the

Berger's approximation. Wah (1963a) used the simplified Berger equation with the assumption that the in plane displacements are very small at the external boundaries. The author used this equation for the large amplitude vibration analysis of rectangular plates with various boundary conditions. The author (1963b) also applied the Berger equations for circular plates. Schimdt (1974) developed a new perturbation method for the analysis of circular plates and membranes with finite axisymmetric deflections. The resulting differential equations are reported to be same as the Berger's equations.

1.2.2.2 Nonlinear vibration of rectangular plate

Previously, the development of nonlinear plate theories is discussed. Now, different methods applied to rectangular plate vibration problems are reviewed. Srinivasan (1965) applied the Ritz-Galerkin technique to obtain the nonlinear free vibration response of hinged-hinged beams and plates for different boundary conditions. Stanišić and Payne (1968) introduced a technique based on the Galerkin approach for determining the natural frequencies of rectangular plates with discrete masses added, considering simply supported and clamped boundary conditions. Their results indicated that the natural frequencies decrease with added mass, whereas the deflections and stresses of the plate tends to increase. A lumped parameter model for large deflection of a rectangular plate was developed by Bayles et al. (1972) using the finite difference method. A dynamic analogue of the von Kármán theory was used. Results were compared with that of Yamaki (1961) and found to be accurate enough when the deflection of the plate is less than its thickness. Next, the calculation of nonlinear natural frequencies of the beams and plates subjected to large amplitude free vibrations was presented by Mei (1973) using the Finite element displacement method. In this study, the nonlinearity was considered due to large deflections. The author derived the stiffness matrix formulation for a plate element based on a modification of the Berger’s hypothesis. As a result, the nonlinear behavior of the hard spring type was shown clearly as the dimensionless amplitude increases. Vendhan (1975) considered the Berger equation for the nonlinear vibration analysis of elastic plates. Satyamoorty and Pandalai (1979) derived the large amplitude flexural vibration of orthotropic skew plates using the Berger’s approximation. In Leissa's monograph (1993) the Berger’s technique was extended to include the vibrational behavior of these nonlinear plates. This approach assumed a solution based on the spatial modes and some temporal function. Dumir and Bhaskar (1988) have used the point collocation method to study the large amplitude vibrations of orthotropic rectangular plates. Plates with all edges clamped or simply supported and fixed edges have been considered in this work. Han and Petyt (1997) studied the nonlinear vibration analysis of thin, isotropic rectangular plates using the Hierarchical finite element method (HFEM). They employed the von Kármán nonlinear strain-displacement relationship to formulate the mathematical model and the harmonic balance method was used to obtain the eigenvalue-like equations. A modified form of the Berger’s hypothesis was also employed to study the in-plane membrane force averaging effect on the geometrically nonlinear behavior. The influence of large vibration amplitude on the frequency and mode shape of a fundamental mode was presented and the results were compared with other results from the published literature. A new methodology that could be employed for the plate structure problems with any combination of boundary conditions was proposed by Saha *et al.* (2005) to determine the nonlinear natural frequencies and mode shapes. In their study, the static analysis served as a basis for the subsequent dynamic problems, and both these problems were formulated through the energy method. The solution methodology was employed as an iterative numerical scheme using the technique of successive relaxation.

1.2.2.3 Nonlinear vibration of circular plate

Many researchers have studied the circular plate though their number is smaller compared to the rectangular one. Large amplitude free vibration of a circular plate was studied by Srinivasan (1965), using the Berger's approximation. Pal (1970) investigated the forced vibration of circular plates subjected to aerodynamic heating using the same approximation. Lee et al. (1971) have indicated that the errors generate from the Berger's hypothesis, for a clamped circular plate, depend on the Poisson' ratio and the ratio of radius to thickness of the plate. They also found that the errors are minimized as the Poission's ratio increases. Huang and Meng (1972) investigated the large deflection of non-uniform circular plates using the Berger's approximation. The Ritz-Kantorovich method was used to decouple the von Kármán equations. Although the assumptions for existence of harmonic vibrations contradict the inseparability of modes in the von Kármán equations, the method is useful for calculating the nonlinear frequencies and mode shapes very easily. Ramachandran (1974a) analyzed the large deflection of circular plates having some part simply supported and other part clamped. Energy method was used to solve the problem. The results were in good agreement with that of Wah (1963b). Using the Berger's approximation, the author (Ramachandran; 1974b) analyzed the large deflection of circular plate having elastic restraint boundary conditions. The author (Ramachandran; 1975) also studied the large amplitude free flexural vibrations of circular plates with linearly varying thickness using the same approximation of Berger. Huang and Al-Khattat (1977) showed that, for radially restrained circular plates, the Berger's approximation is accurate at low amplitudes but the errors increase as the amplitude increases. They also found that the Berger approximation is not applicable for plates with movable edges.

1.2.3 Sound radiation from rectangular plate

Sound radiation from plate structures remains a problem of practical interest as a lot of machinery housing in the industry can be represented to such shapes. The plate analysis can be done for two cases: baffled or unbaffled. A baffled plate represents a plate vibrating in the plane of an infinite rigid surface. The rigid baffle allows us to transform the double layer integral representation into a simple one and it renders the vibro-acoustical problem solvable. Although this type of plate doesn't exist in reality, a study of this type of plate is of importance as it provides the simplest way to calculate the approximate sound radiation from the plate structures.

Much of the work has been done on the estimation of sound radiation from baffled and unbaffled plates. Usually, a simply supported baffled plate is one of the simplest models to predict the radiation behavior of a plate. Maidanik (1962) first statistically investigated a single mode radiation resistance of a finite simply supported baffled plates. The author suggested that the effect of the attachment of stiffeners to a panel is effective to produce many smaller panels. Davies (1971) approximately evaluated the modal coupling coefficients, for both low and high frequencies, from an infinite set of linear simultaneous equations of the plate modal velocity amplitudes in terms of modal admittances of the plate-fluid system. But the analysis was based on the in vacuo modes of the plates. The effective amount of coupling and hence the effective radiation damping acting on a mode depends on the relative magnitudes of the structural damping i.e., on the widths of the modal resonance peaks, and the frequency spacing of the resonances. Wallace (1972) first theoretically obtained an expression for the real part of the radiation coefficients versus frequency only for diagonal terms, and defined it as radiation efficiency to compare the sound radiation from different modes. However, the asymptotic solutions are only valid in the low frequency region. Leppington et al. (1981) analytically evaluated the acoustic radiation efficiency for higher modes of a simply supported panel near the coincidence frequency. Williams and Maynard (1982) used the computationally efficient Fast Fourier Transform (FFT) method to find the pressure and particle velocity for plane baffled radiators. However, at low frequencies, convergence problems were found when a finer resolution of sample spacing was used to calculate results for higher order modes. Subsequently, Williams (1983a) showed a novel approach to compute the acoustic power based on the MacLaurin series expansion of the Fourier transformed velocity. Thus, the acoustic power radiated by monopole, dipole, quadrupole and other multipoles could be easily identified. This method is applicable for both baffled and unbaffled plates. Berry et al. (1990) gave a general formulation for sound radiation from rectangular baffled plates using the algebraic polynomial functions. The elastic constants at the edges were varied to reproduce the simply supported, guided, clamped and free boundary conditions. Although a wide range of boundary conditions can be handled by this formulation, the method yields error in high frequency region due to the use of less accurate polynomial functions. Later, Berry (1994) extended the previous approach by considering the fluid loading over the plate. The Green's function was expanded using the Taylor's series to provide a direct decomposition of the radiator's power into multi-poles of radiation. In this way, the singularity of using Green's function is avoided and an analytical method to compute the radiation impedance coefficients is developed. Xie et al. (2005) computed the averaged radiation efficiency of simply supported plate and strip in forced vibration using the modal summation approach. Zhang and Li (2010) computed the sound radiation from a baffled rectangular plate with arbitrary boundary conditions. They used the modified 2-D Fourier cosine series substituted by several 1-D series. The unknown Fourier coefficients are then determined by the Rayleigh-Ritz method. The sound radiation obtained from the radiation resistance matrix was compared with the other published numerical results. This method is advantageous because the boundary conditions can be easily changed and it is analytically very accurate.

Although a baffle plate approximation simplifies the actual condition and it is also valid in the high frequency region, this approximation is no longer valid in the low frequency region. Williams (1983b) extended the FFT method to determine the sound radiation from unbaffled thin circular plates. However, the same convergence problem was found in here also like that of baffled plate (Williams, 1982). Atalla et al. (1996) were the first to study analytically the sound radiation from unbaffled plates with general boundary conditions. However, the authors neglected the pressure jump during normal velocity calculation. Oppenheimer and Dubrowsky (1997) experimentally calculated the sound radiation of an unbaffled plate by the comparison of impact data between the balls and a plate, and an empirical formula was generated. In that approach, the baffled plate results are actually modified in the corner and edge mode regions. The improvement in result from the baffled plate is more at low frequencies. Laulagnet (1998) analytically calculated the sound radiation from an unbaffled plate with simply supported boundary conditions and submerged in light or heavy fluid, without neglecting the pressure jump in the plate velocity. Langley et al. (2006) used a different method to numerically compute sound radiation from planar structures using wavelets. Baffled, unbaffled and partially baffled plates can be studied very efficiently by this procedure. This method is computationally faster than the Boundary Element Method (BEM) though it is only applicable for planar radiators.

1.2.4 Damage detection techniques

Cracks pose a problem in the normal operation of the structures. A fatigue crack can develop in a vibrating structure, which expedite its inadvertent failure. So, the methods allowing early detection and localization of the cracks are of the utmost necessity for smooth running and longevity of machines and structures. Nondestructive damage identification methods can be categorized as either local or global damage detection techniques based on four steps of damage detection: existence, localization, extent and prediction. Local damage identification techniques such as, ultrasonic, X-ray and eddy current based methods are suitable for obtaining the exact localization and extent of the damage. However, they require that vicinity of the damage should be previously known and the damaged area is readily accessible for testing, which may not be possible for large structures. Vibration based damage detection techniques evoke these limitations and that's why these techniques are gaining importance as global damage detection techniques. These types of damage identification methods can also be classified as "Model based damage detection" and "Response based damage detection". The model based methods require a detailed numerical model in case of a complex structure or some approximate analytical model (if available) in case of a simple structure. The response based methods depend only on the experimental response data from the structures. Now a days, the model based methods are gaining importance as they are cheaper than the costly experiments and are also helpful in the prediction of damages.

A comprehensive review of the vibration-based damage identification methods have been reported by several researchers (Doebling et al., 1996; Sohn et al., 2003; Carden and Fanning, 2004; Frisnell, 2007; Fan and Qiao, 2011). The identification methods basically rely on the change in modal parameters (natural frequencies, mode shapes, and modal damping) which are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, a change in such physical properties, for example a reduction in stiffness results from the onset of a crack, will cause detectable changes in the modal properties.

Major components in civil, aerospace, and mechanical engineering can be approximated to a plate or a beam. However, relatively few references are available dealing with the damage identification methods of plate structures. Cawley and Adams (1979) first shown that the natural frequency can be used as a crack detection parameter. Pandey et al. (1991) used the modal curvature method to localize a crack in beams and plates. Modal curvature was computed using the central difference approximation and the damage localization was done by calculating the difference from the undamaged one. Pandey and Biswas (1994) implemented a new damage detection technique from the flexibility matrix of a structure. Changes in flexibility are higher for lower modes and thus the damage can be detected and localized by monitoring few lower modes only. The authors have tested the procedure on the experimental data collected from wide-flange beam and got satisfactory results. Stubbs and Kim (1994) proposed a damage index based crack detection technique which is based on the change in modal curvatures at different locations. The damage indices for all modes were then summed to result in the damage index. Faults were then localized using some statistical methods. Farrar and Jauregui (1998a, b) compared several methods, such as the damage index method (Stubbs and Kim, 1994), the mode shape curvatures (Pandey et al., 1991) or the change in flexibilities (Pandey and Biswas, 1994), for damage detection in a real road bridge. The author commented that the damage index method performed superior than the other methods. Sampio et al. (1999) extended the modal curvature method of Pandey et al. (1991) to develop a third level of damage detection method for a real bridge. The method uses all frequencies in the measurement range rather than the modal frequencies only. A numerical model was developed using the lumped mass and compared with the experiment. Cornwell et al. (1999) applied a strain energy method to detect the damage in plate-like structures. This method was based on the changes in the strain energy of the structure and required only the undamaged and damaged mode shapes of the structure. The algorithm was found to be effective for localizing 10% stiffness reductions too, using a few modes only.

Several researchers (Rucka and Wilde, 2006; Kim et al., 2006; Fan and Qiao, 2009) have used the wavelet transform based approach for damage detection purpose. It requires the mode shapes of the structure either obtained from a numerical model, simplified analytical model or from the experimental results. The two dimensional wavelet based damage detection method was first addressed by Wang and deng (1999). The authors considered a steel plate with an elliptical hole which is subjected to a uniform tensile loading. The static displacement field obtained from an analytical model was used as input for the Haar wavelet transform and then the crack tip location was found by the variation of wavelet coefficients. Douka et al. (2004) used the one dimensional discrete wavelet transform (DWT) to a rectangular plate having a crack running parallel to one side of the plate. Later, the previous approach was extended for the two dimensional case by Loutridis et al. (2005). Detail wavelet coefficients were used to determine the location, length and depth of the crack. Rucka and Wilde (2006) have applied the one dimensional Gaussian wavelet and two dimensional reverse biorthogonal wavelets for damage detection in beam and plate respectively. Experimental and numerical mode shapes were used for the study. Maximum number of vanishing moments were found to give better results. The modulus and angle of two-dimensional wavelet transform was used for damage identification. Kim et al. (2006) used the multiresolution analysis of two dimensional Haar wavelet for damage localization in a rectangular plate. In this approach, the damage can be effectively detected using lower modes only. Although only numerical mode shapes were used in that study. Fan and Qiao (2009) presented the continuous wavelet transform (CWT) based damage localization of rectangular plates using the Dergauss2d wavelet. The algorithm was tested with the numerically and experimentally obtained mode shapes and its excellent noise immunity was shown with small datasets. They also suggested that the CWT based damage detection methods are excellent singularity scanner.

Forcing point impedance and mobility based crack detection methods are one of the reliable model based methods in structural health monitoring. This approach was first proposed by Nezu (1980) to detect damage in structural member. Bamnios and Trochids

(1995) applied this method to detect transverse crack in a cantilever beam. Later, Prabhakar et al. (2001) also used this technique for crack detection in rotors. Simulated results were found to be more prominent when the forcing point is far from the location of the crack. They also validated the results experimentally in their next paper (Prabhakar et al., 2002). Recently, Kim et al. (2011) have developed a reference free, impedance based crack detection method for plates using the Lamb waves.

All the previous methods of crack detection are based on a linear model and so nonlinearities in the structure are neglected. However, the consideration of nonlinearities become evident when a structure is subjected to large deflection (aircraft wing model) or an accurate prediction is needed (consideration of a breathing crack). Nichols et al. (2003) explored the use of attractor dimension as a damage indicators of a structure. Two ways of detection were proposed as takens estimator and correlation dimension. From the experimental results, the Takens estimator was found to be superior than the correlation dimension. Trendafilova and Manoach (2008) developed two methods of damage identification and localization in a thin rectangular plate subjected to large amplitude vibrations using a state space representation of temporal data sets. The first one is based on representing the state space points on the attractor of the structure and the other one on the Poincaré maps. Later, Manoach et al. (2012) extended the previous approach of using the Poincaré map for damage detection of a laminated beam. A numerical model was developed and a change in temperature was also included. The authors have shown that the temperature variation substantially affects the Poincaré map. Andreaus and Baragatti (2012) proposed a nonlinear damage detection method for beams based on phase plots. Introduction of a fatigue crack was found to distort the phase plots. Here, the crack closing effect is the main source of nonlinearity in the system and for this reason, the bilinear frequency approximation (Shaw and Holmes, 1983) appears to be a promising one. Both the experimental and numerical validations were given.

The main disadvantages of vibration based health monitoring is that the signal must be acquired with contact sensors which may not be possible in a hazardous environment. Recently, Arora et al. (2014) introduced a new non-contact, acoustic based damage detection method to detect the location and extent of the damage. Some microphones were mounted close to a fixed-fixed plate and the acoustic excitation was provided by a loudspeaker. The authors assume that the pressure signal at the microphones are proportional to the velocity of the plate. Hence, they have neglected the use of acoustical holography theory. Later, the mode shapes were evaluated from that signal. The damage location and extent were evaluated by the flexibility method of Pandey and Barai (1995) analyzing lower modes only.

1.2.5 Vibrations of rectangular cracked plates

Numerous papers are available on the static analysis of cracked structures, though much of its dynamic characteristics are unknown till date. Some reasons are the inherent crack tip nonlinearity, friction and the intermittent contact between the crack faces. Several numerical methods are available to analyze this problem. However, the main hurdle of performing dynamic analysis is the use of huge computational resources. That’s why several approximate analytical methods were developed in the past to ease some complicated problems as, a surface crack in a rectangular plate (Rice and Levy, 1972). Dimarogonas (1996) reported a comprehensive review on the vibration of cracked rotors, bars, beams and plates with an extensive collection of references. Now, the literatures concerning the rectangular cracked plates are shown in two parts viz. Numerical and Approximate analytical approach.

1.2.5.1 Numerical approach

Several numerical methods were used in the previous literatures to analyze the problem either in static or dynamic domain. Some popular techniques are the Finite Element Method (FEM), Boundary Element Method (BEM), Boundary collocation method and the Ritz method. Every method mentioned here has some advantages and disadvantages. So, the choice of a method fully depends on the accuracy or the complexity of the problem in hand. Here, the literature review of a cracked plate is described in two separate paragraphs; first a few static problems are discussed followed by, its implementation into dynamics.

In Linear Elastic Fracture Mechanics (LEFM), the exact formulation of a cracked plate is done by considering stress singularity at the crack tip. It is found in the literature that the stress singularities of sectorial plates are studied prior to the study of cracks. This is because when the sector angle tends to 360^0, it generates the usual crack tip singularity. Williams (1952) first investigated the stress singularities due to the boundary conditions in the angular corners of isotropic thin plates under bending. However, two dimensional plain strain state was assumed in the derivation. First three dimension model of a cracked plate was analyzed by Raju and Newman (1979) using the finite element method. Circular or penny-sharped cracks and internal elliptical crack were considered in their study. Singularity at the crack tips were included in the model. The authors have shown that the calculated stress intensity factors are very close to the exact solution. Following the same way, Alwar and Nambissan (1983) calculated the stress intensity factors of the cracked plates subjected to bending. Singular brick element was used for the analysis. The authors have reported that the stress intensity factors obtained in this model is higher compared to a same two dimensional model. Wang et al. (2003) used the boundary collocation method to compute the stress intensity factors at the tip of a single or multiple cracks under unidirectional tensile load. The curvature of plate edges were considered in the study. The authors have shown that their method is better applicable for shorter cracks. The stress intensity factors of finite plate were found to be larger than infinite plate due to more interaction between the plate edges and cracks in the former case.

The vibration of a cracked plate was first investigated by Lynn and Kumbasar (1967) using a Green's function approach. They used the method of Bogoliubov and Mitropolski (1961) to solve the Fredlom integral equation of the first type. The integral equation was shown to satisfy the edge condition around a narrow crack. Petyt (1968) analyzed the vibration of a rectangular plate having a fatigue crack. The plate was also subjected to a tensile load due to the hoop stress of the aircraft fuselage. Mainly experimental observations were reported along with some theoretical calculations. The finite displacement method and the triangular elements were chosen to understand the buckling of crack, which was also compared with experimental results. The authors found that an increase in width decreases the amplitude in the crack region due to an increase in curvature around it. Keer and Sve (1970) was the first to analyze the vibration of cracked plate subjected to bending. An external crack, two collinear cracks and an internal centre crack were studied by the authors. Due to symmetry of the crack position, the problem was reduced to a dual series and thus became easier to solve. Westmann and Yang (1967) modified the previous method and obtained the solution for simply supported and clamped boundary conditions in terms of the second type Fredlom equations. They found that the strain energy release rate is smaller for the clamped boundary conditions and hence suggested its use to minimize the fracture. Stahl and Keer (1972) presented an analogous method to find the natural frequencies of a plate with a side crack or a centrally located internal crack. Stress singularity was considered at the crack tips. The results were very accurate but the procedure of the model involved same disadvantages as Keer and Sve (1970). Later, Aggarwala and Ariel (1981) followed the same procedure to solve the eigen problem of simply supported rectangular plates. The authors pointed out that the frequency in case of a side crack appeared to be higher compared to an inner crack. Both Hirano and Okazaki (1980) and Neku (1982) analyzed the free vibration problem of a cracked plate using the Levy type plate theory (i.e., two parallel edges are simply supported and arbitrary boundary conditions on the other two sides). The unknown deflection and slope due to the crack was included in a integral equation and later expanded to a Fourier series. A finite fourier transform was applied to get the natural frequencies. Solecki (1975) attempted to generalize the previous model to analyze an arbitrary located crack in a rectangular plate. Later, the author (Solecki, 1983) used a combination of the finite Fourier transformation of discontinuous functions and the generalized Green-Gauss theorem to study the natural frequencies of a simply supported plate with an arbitrarily located crack parallel to one of the plate edges. Maruyama and Ichinomiya (1989) presented the experimentally obtained natural frequencies and mode shapes of edge and internal cracked clamped rectangular plates. The natural frequency of an internally cracked plate was found to increase up to 60^0 and then decrease. They found that the decrease in natural frequency, for a side crack, is more when it emanates parallel to the longer side instead of the shorter side. Qian et al. (1991) used the finite element method to find the eigenvalues of a finite rectangular plate having a centrally located through crack. The stress intensity factors were used to modify the stiffness matrix through the calculation of strain energy. Thus, adequate meshing at the crack tips become unimportant which subsequently decreases the computation time. The results were also in good agreement with the result of Solecki (1983). Following the same way, Krawczuk (1993a,b) analyzed the natural frequency variation of a centre cracked plate with crack length and position. Simply supported and cantilever plates were considered for analysis. The results were validated with previous numerical results. The authors concluded that the natural frequencies vary differently for various mode shapes instead of the same boundary condition, crack length and position. Later the same methodology was extended to the forced vibration problem (Krawczuk, 1994; Krawczuk and Ostachowicz, 1994) to study the effect of crack length and position on the vibration amplitude of cracked cantilever plates. The authors (Krawczuk et al., 2001) further extended the study by considering plasticity at the crack tips. Liew et al. (1994) employed the domain decomposition technique along with the Ritz method to determine the upper bound of natural frequencies for cracked plates. However, the displacement and slope continuities were not satisfied at every point along the interconnecting boundaries. So, the crack tip stress singularity was not properly generated. Ramamurti and Neogy (1998) used an extended Rayleigh-Ritz method to get an approximate solution for the natural frequencies of a cracked cantilever plate. Although the main motivation of their research was to test the natural frequency variation as a potential crack detection method, a nonrotating plate was considered for the analysis. The authors later argued that natural frequency is not a good indicator for damage. Su et al. (1998) applied the extended two level finite element method for free vibration of a centre cracked plate. The main advantage of the model is that the arbitrary boundary conditions can be studied. The results for simply supported boundary conditions were also compared with the results obtained by Stahl and Keer (1972) and an excellent agreement was achieved. Ma and Huang (2001) experimentally obtained the natural frequencies and mode shapes of a cantilever plate having a side crack. The results were compared with that obtained from numerical simulation in ABAQUS. The author commented that the mode having out of plane displacements are very dangerous for a structure. The crack closing phenomenon was found to be important for in plane vibration where the stress intensity factor is very low. Fujimoto et al. (2003) analyzed the free vibration characteristics of a centrally cracked plate subjected to a constant tensile load. The Hybrid finite element method was used for eigenvalue extraction and the Body force method was used for in-plane stress analysis. They have found that an increase in applied tensile load also increases the natural frequency irrespective of any change in mode shapes. Crack buckling was pointed out to be an important phenomenon as it affects the natural frequencies and mode shapes severely at a small tensile load. Amabili (2006) theoretically and experimentally investigated the large amplitude vibrations of rectangular plates with geometric imperfections subjected to harmonic excitation. It was observed that the geometric imperfections which produce softening type nonlinearity in small amplitudes can behave as hardening type nonlinearity at large amplitudes. Huang et al. (2009) analyzed the free vibration of thin square plates having side cracks using the Ritz method. The stress singularities were introduced by some set of corner functions. Simply supported and free boundary conditions were used and some results were verified with that of Stahl and Keer (1972). However, the method uses a lot of algebraic polynomials to converge and also suffers from a convergence problem for larger cracks. Following the same way, the authors (Huang et al., 2011a) extended the previous approach to vibration of thin plates having internal cracks. Corner functions are used for two crack tips and the plate is subdivided into several regions. Natural frequencies and mode shapes were shown for simply supported and free boundary conditions. The results were also compared with the published literatures. Later, the authors (Huang et al., 2011b) applied the previous method to thick rectangular plates with arbitrary oriented cracks. A decrease in the natural frequency with an increase in the crack angle was reported for the simply supported and cantilever plates having internal through cracks. Saito et al. (2008, 2009) investigated the natural frequency veerings of a cracked cantilever plate using a finite element model. Then nonlinearity due to crack closing phenomenon was introduced and solved by a hybrid frequency/ time domain method. The results were also compared with that obtained from the extended bilinear frequency approximation, which was previously proposed by Shaw and Holmes (1983). The bilinear frequency term appears due to intermittent contact of two crack faces and this approximation found to be a good estimator of damage than the system's natural frequency, as reported by Andreaus and Baragatti (2012). Saito and Epureanu (2011) extended their previous work (Saito et al., 2008 & 2009) into the reduced order model of a turbine blade. Contact nonlinearity due to delamination or crack is studied using the bilinear mode approximations. Bachene et al. (2009) analyzed the free vibtation of rectangular plate having a side crack or a centre crack parallel to one edge of the plate. Extended finite element method (XFEM) was used for the problem. The effects of shear deformation and rotary inertia were taken into account based on the Mindlin's plate theory. The reduced integration with nine node elements was found to be in good agreement with the accurate result of Stahl and Keer (1972). Natural frequencies were observed to decrease more for antisymmetric modes. The author commented that the XFEM is an efficient method for the dynamic analysis of plates containing discontinuities.

1.2.5.2 Approximate Analytical approach

Although the numerical methods are accurate, implementation of these numerical methods becomes cumbersome for complicated problems (for example, a surface crack). To tackle this type of problem with sufficient accuracy different approximate models can be found in the literature. Here the developments of static models are presented first, followed by their implementation in vibration problems.

Irwin (1962) was the first to examine the stress field of an elliptical crack, with the approximation of a part-though crack. The author also derived a relation between the surrounding stress field and the crack extension force. Gross et al. (1964) calculated the stress intensity factors of single edge bend specimen. The authors used the Boundary collocation method to the William's stress function for that purpose. The results were found to be accurate for small ratio of crack length to specimen width. Okamura et al. (1969) obtained the lateral deflection, the load carrying capacity, and the stress intensity factors of a rectangular cross section single-edge cracked column with hinged ends under compression. They compared an un-cracked column with a cracked column for different parameters, and examined the effect of a crack on load carrying capacity and deflections. The effect of compliance due to bending was considered in their formulation, but ignored the effect of compliance due to rotation induced by the axial load. Rice and Levy (1972) represented a part-through surface crack as continuous line springs which is also famous as the Line Spring Model (LSM). This approximation reduces a complicated three dimensional problem to an equivalent two dimensional problem which abruptly decreases the lengthy computation. The compliance coefficients of the springs were chosen to match those of an edge-cracked strip in plane strain. The authors observed that the model is not valid for very small crack (crack length to height ratio must be greater than 0.25). The model also assumes that the crack length varies slowly from the free surface. Later, several researchers have used it in different problems and with the advent of time this model is subsequently modified. Delalae and Erdogan (1981) improved the line spring model by considering the transverse shear deformation of the plate. Reissner plate theory was used to formulate the problem and then solved by the boundary integral method. Later, this model was extended to antisymmetric conditions (Joseph and Erdogan, 1991). King (1983) simplified the line spring model to analyze an elasto-plastic surface crack. The simplification assumes a crack of constant depth, though it greatly eases the computational effort of using the couple integral equations of Rice and Levy (1972). The model was shown to be applicable when the crack length to thickness is more than one. Later, Zheng and Dai (1994) have applied this model to a biaxially loaded rectangular plate having an angled part-through surface crack. The stress intensity factors were calculated and compared with the exact solution. Wen and Zhixe (1987) investigated the relationship between the stress intensity factors derived from the LSM using the Reissner plate theory and the Kirchhoff plate theory. A modified line spring model was also suggested by them which neglect the local effect of deformation. Zeng et al. (1993)

implemented the LSM in the boundary element method to investigate the stress intensity factors of a surface crack, which were later post-processed by the virtual crack extension technique to obtain the corrected value of stress intensity factors. The values were shown to be in good agreement with the accurate solution. Cordes and Joseph (1995) used the Reissner plate theory along with the LSM to calculate the stress intensity factors in a surface or internal crack which is subjected to residual stresses too. These authors presented a series of results for different crack lengths and depths. The results were also found to be close with that obtained from the classical theory of LSM (Irwin, 1962) and the finite element model. However, the error was found to increase as the order of the loading increases. Liu et al. (1999) analyzed the surface crack problem in thick plate and shell using the boundary element method with mixed boundary conditions. The LSM was used for the formulation of crack terms and then the J-integral was evaluated up to large scale plastic deformation. The solution obtained from the LSM was found to be in good agreement with the result of previous literatures in elastic and moderately plastic domain, though it was invalid in case of the large scale plastic deformation. Goncalves and Castro (1999) studied the semi-elliptic surface crack and quarter-circular corner crack using the LSM and also discussed its implementation in the commercial FE software (ABAQUS). The stress intensity factors was calculateded for pure tension and bending. The authors have also shown that the deviation of LSM from the accurate solution increases where the crack opens to the free surface or, where the crack depth varies rapidly. Large deviation was reported for the pure bending load and large crack depth to thickness ratio. Therefore, from the above discussions it's clear that the LSM is very effective for the elastic domain analysis, though it has some drawbacks when the large scale plastic deformation and crack-surface interaction is considered.

Although several researchers have used the LSM in static domain, its use in dynamic domain is very less. Khadem and Rezaee (2000a) first introduced that model for crack detection in a simply supported plate by using the modified comparison functions. However, the author neglected nonlinearity in the model and only the effect of bending compliance was considered. The results suffered from unacceptable errors at some frequencies. Later the authors (2000b) established an analytical approach for crack detection in a rectangular plate under the application of external load and different boundary conditions. They found that different crack depths and positions affect the natural frequencies in different ways. Wu and Shih (2005) have investigated the dynamic characteristics and natural frequencies of an edge cracked plate subjected to inplane load. The von Kármán plate theory was used and the Matheiu equation was solved by the incremental harmonic balance method (IHB). The authors have observed that the static component of the inplane load decreases the natural frequency. They also found that the increasing crack length ratio and the plate aspect ratio shrink the instability region, whereas an increase in the dynamic component of inplane load increases the instability. Using the Berger's (Berger, 1955) model and the LSM of Rice and Levy (1972), Israr et al. (2009) presented an approximate analytical solution of the thin rectangular plates with a centrally located part-through surface crack subjected to large amplitude vibration. The authors related the far field tensile and bending stress with the crack tip tensile and bending stress respectively but avoided the coupling between them. This coupling is evident due to the eccentricity of the ligament with respect to the centerline of the plate thickness (Rice and Levy, 1972; King, 1983). Ismail et al. (2012) extended the previous model (Israr et al., 2009) to a plate with an angled crack. The authors found that the natural frequency of the cracked plate increases with an increase in crack angle. They also identified the natural frequency of a plate with a vertical crack is the same as that of an intact plate.

1.2.6 Vibration of cracked circular plates

In literature, articles on vibration of cracked circular plates are very less compared to its rectangular counterpart. Lee (1992) used the Rayleigh method with simple sub-sectioning technique to obtain the fundamental frequencies of annular plates having internal concentric cracks. Results are presented for simply supported and clamped boundary conditions. Using the eigenfunction expansion method of Williams (1952), Huang et al. (1993) discussed the singularities of moments and shear forces at the apex of a sector plate with simply supported radial edges in an exact solution for vibrations of such a sectorial plate. Free vibration of circular plate with V-notches was first analyzed by Leissa et al. (1993) using the Ritz method. Algebraic-trigonometric polynomials and some special functions called “corner functions” are used to describe the displacement of the plate. The corner functions generate the stress singularities properly and also accelerate convergence of the results. Yuan et al. (1994) presented a natural frequency estimation approach of a circular or annular plates having a radial or circumferential through crack using the Ritz method. The method assumes several sector elements which are joined together by artificial springs. The natural frequency of the first mode was only compared with the published literature results. Anifantis et al. (1994) performed the vibration analysis of an annular plate having a surface peripheral crack, where the crack was modeled as local reduction of rotational stiffness in a beam-like structures. Natural frequency was found to increase with increasing stiffness. Ramesh et al. (1997) experimentally investigated the free vibration of an annular plate with periodic cracks. Inner side of the annular plate was fixed by some bolts. Both the hammer and shaker table test were carried out. Crack emanating from the outer boundary was found to decrease the natural frequency more than a crack emanating from inner boundary. The author commented that the resonance frequencies are more for the circumferential modes than the diametral modes. Huang and Ma (2000) experimentally investigated the free vibration response of a circular plate having an edge crack. Experimentally obtained natural frequencies and mode shapes were validated by the FEM simulation. Stress intensity factors of the out-of-plane modes were found to be more than the in-plane modes. Thus the authors commented that the crack propagation may occur for the out-of-plane modes. Wang (2002) investigated the free vibration of circular plate weakened along a concentric circle considering clamped and simply supported boundary conditions. The author reported an optimum radius of concentric circle, in case of clamped boundary conditions, where the frequency of circular plate remains unchanged. Yu (2009) extended the previous approach to free and movable boundaries. They commented that the movable boundary decreases the fundamental frequency greatly than a free boundary. Rao and Rao (2013) extended the previous method to elastic translational restraint boundary conditions. Frequency parameter variation with different weakened circle radius were presented. Demor and Mermertas (2008) developed a finite element model for free vibration analysis of an annular plate having a circumferential crack. A trapezoidal element was used in the element as straight crack.

1.3 Summary of literature review

In the previous section, a broad review of available literatures is performed. Both the linear and nonlinear plate theories are discussed for thin plates. The Kirchhoff's theory is found to be well applicable for the small deflection of plates. For large deflection analysis, the plate theory of von Kármán (1910) and Berger (1955) are widely used. The von Kármán's plate theory can provide an accurate result, though it is too tedious in implementation and not accurately applicable to some complicated problems. On the other hand, the Berger's theory is attractive due to its simplicity in handling of complex problems, though a little sacrifice of accuracy is always involved. Literature involving the sound radiation from rectangular plates is reviewed next. Some researchers have used a statistical tool (Maidanik, 1962), analytical methods (Wallace, 1972; Leppington et al., 1981; Laulagnet, 1998) or fast fourier transform (Williams and Maynard, 1982; Williams, 1983a & b) to explain the sound radiation characteristics of different modes. A few of them have tried to generalize the previous approaches to handle arbitrary boundary conditions (Berry et al., 1990; Berry, 1994; Zhang and Li, 2010). Till now, the presented literatures deal with only uncracked plates (with the assumption that there is no flaw in the material). Several linear domain based crack detection methodologies are also reviewed. These are based on change in mode shapes (Pandey, 1991; Pandey and Biswas, 1994), by the application of wavelets (Rucks and Wilde, 2006; Kim et al., 2006; Fan and Qiao, 2009) and change in impedance (Baminos and Trochids, 1995; Kim et al., 2011). However, these methods are not accurate enough when significant nonlinearity is present in the system. In this situation, use of nonlinear methods such as, change in attractor dimension (Nicholas et al., 2003), Poincaré map (Trendafilova and Manoach, 2008;

Manoach et al., 2012) and bilinear frequency approximation (Andreaus and Baragatti, 2012) can solve the problem. For monitoring in a non-contact way, acoustic based methods (Arora et al., 2014) can also be used. However, these monitoring methods cannot provide a detailed insight of the structure. So, vibration analysis of cracks is very important for further prognosis. Some researchers have used the Finite element method (Saito et al., 2009; Saito and Epureanu, 2011; Bachne et al., 2009) or, the Rayleigh-Ritz method (Leissa et al., 1993; Huang et al., 2009; Huang et al., 2011) for that purpose. For some complicated problems, an approximate analytical analysis is also performed (Wu and Shih, 2005; Israr et al., 2009; Ismail et al., 2012). Some exact solution is available for simple problems, as a concentric crack in a circular plate (Wang, 2002; Rao and Rao, 2013). Moreover, it is observed in the analytical formulations the governing equation is kept unchanged when the exact modal function is available (Huang et al., 2009; Wu and Shih; 2005; Rao and Rao, 2013), though it is modified in some cases when the exact modal function is not available (Israr et al., 2009; Ismail et al., 2012).

1.4 Research objectives

Motivated by the issues arising out of the literature survey, the objectives of this research are to develop a simple yet accurate linear and nonlinear models which will be advantageous for either vibration or acoustic based crack detection process. In particular, they are summarized as follows:

- To develop a linear model for vibration based crack detection of a rectangular plate.
- To extend the linear model for acoustic based crack detection of a rectangular plate.

- To develop a nonlinear model for vibration based crack detection of a rectangular plate.
- To develop a nonlinear model for vibration based crack detection of a circular plate.

1.5 Dissertation Outline

The remaining part of the dissertation is arranged as follows:

In Chapter 2, a linear model for a side cracked rectangular plate is developed for fault diagonosis and mobility monitoring method is proposed. The result is further validated with the simulated result of ABAQUS. Results are shown for different crack lengths, angles and positions.

In Chapter 3, The previous model is further extended for acoustics based crack detection, which is validated with LMS Virtual Lab.Acoustics. Radiation efficiency and sound power graphs are presented for various crack lengths, angles and positions.

In Chapter 4, description of the experimental set-ups and procedures are mentioned. Experimentally obtained natural frequencies and mode shapes are also documented.

In Chapter 5, a nonlinear model of a surface cracked rectangular plate is considered. Different amplitude and phase curves are also shown for three different boundary conditions and with various crack lengths, angles and positions. Finally, crack detection is proposed using phase plots.

In Chapter 6, a nonlinear model of a cracked circular plate is developed. Here also, various amplitude and phase curves are plotted for different crack parameters. For crack detection purpose phase plots are used.

The conclusions of the thesis are presented in Chapter 7.