CHAPTER 1

INTRODUCTION

1.1 Basic and Historical review of Thermoelasticity:-

Most materials undergo appreciable changes in volume when subjected to variations in temperature. If the thermal expansions or contractions, due to temperature variations, are not freely admitted, some internal forces develop which are called thermal stresses. Conversely, a change in volume is attended by a change in temperature. When a given element is compressed or dilated, the volume change is accompanied by heating or cooling. The domain of thermoelasticity encompasses the study of the influence of the temperature of an elastic solid upon the distribution of stress and strain, and of the inverse effect of the deformation upon the temperature distribution. In other words, thermoelasticity describes the behavior of elastic bodies under the influence of non-uniform temperature fields.

In various fields of engineering sciences specially in the design of nuclear reactors, air-planes, missiles, stream turbines, internal combustion turbines, reciprocating internal combustion engines, high pressure gas compressors and machines which deal with heat conversion, the thermal stress distributions play a very important role. A knowledge of the distribution of thermal stresses is essential to the designers for studying the strength of the material affected by the temperature distribution.

Generally, the change of body temperature is caused not only by the external and internal heat sources but also by the process of deformation itself. Under normal conditions of heat exchange, the heat flux produced by the deformation gives rise to unsteady heating. The change of temperature being small, the corresponding terms in field equations and coupling terms in heat conduction equation; are neglected in the Classical Dynamical Uncoupled theory of thermoelasticity. This theory has two shortcomings. The first and most obvious one is that the elastic deformation has no effect on the temperature field, and the other is that the theory predicts an infinite speed of propagation for heat waves as well as for mechanical disturbances.

Duhamel [20,21] postulated the influence of temperature on the elastic bodies and gave the stress-strain relations. Neumann [54] also gave a stress-strain-temperature relations, similar to Duhamel, which are so-called as Duhamel-Neumann's relations.

Biot [6] in 1956 derived the equations of the coupled theory of thermoelasticity by correcting the equations of motion, which give rise to the Coupled equations, thus taking care of the first shortcoming. But the heat equation, in the Coupled theory, still remains parabolic and hence predicts an infinite speed of propagation for heat waves and mechanical disturbances, which is a physically unreasonable result.

To take care of this paradox, two theories of generalized thermoelasticity are proposed which allow a finite velocity for the propagation of thermal and mechanical disturbances:-

a) **Lord and Shulman** [40] introduced the theory of generalized thermoelasticity for an isotropic body based on a new law of heat conduction, including both the heat flux and its time derivative, replacing the conventional Fourier's law. This theory takes into account the time needed for acceleration of the heat flow, which is known as relaxation in time or second sound effect. The heat equation associated with this theory is a hyperbolic type and hence, eliminates the paradox of infinite speeds of propagation inherent in both the Uncoupled and the Coupled theories of thermoelasticity.

b) **Green and Lindsay** [25] presented a theory of thermo-elasticity with certain special features that contrasts with the previous theory having a thermal relaxation time. In Green and Lindsay's theory Fourier's law of heat conduction is unchanged whereas the classical energy equation and Duhamel-Neumann's relations are modified, by introducing two relaxation times in the governing equations in place of one relaxation time in Lord and Shulman's theory.

One of the important differences between the two theories is that, the energy equation of Lord and Shulman's theory depends on both the strain velocity and strain acceleration whereas the corresponding equation of the Green and Lindsay's theory depends only on the strain velocity.

Green in [26] supplemented the Green and Lindsay's theory and proved the uniqueness of the equations and also studied the propagation of acceleration waves.

The Lord and Shulman's theory was extended by Dhaliwal and Sherief [19] to include both the effects of anisotropy and the presence of heat sources.

The uniqueness of the solution of Lord and Shulman's equations for the isotropic case was studied by Ignaczak [33,34] and for the anisotropic case by Sherief and Dhaliwal [65].

Most often the solutions obtained using Lord and Shulman's theory differ little quantitatively from those obtained using either the Coupled or the Uncoupled theories, though, the solutions differ qualitatively. However, for many problems steep heat gradients, and when short time effects are sought, this theory gives markedly different values than those predicted by any of the other theories. This is the case encountered in many problems in industry especially inside nuclear reactors where very high heat gradients act for very short times.

The main difficulty encountered in solving problems of Coupled or generalized theory of thermoelasticity is that of inversion of the Laplace transforms used. This is mainly due to the fact that the contour integral of Laplace transform's complex inversion formula contains complicated branch points in its integrand.

Among the methods used to invert the Laplace transforms in thermoelastic problems is the method of asymptotic expansions valid for small values of time. Hetnarski [29,30] used this method to obtain the solution for a thermal shock half-space problem and to solve a spherically symmetric problem with a point source of heat, both in the context of Coupled thermoelasticity. For generalized thermoelasticity Sherief and Dhaliwal [66] and Sherief [67] used the same method to solve a thermal shock problem and to obtain the fundamental solution for the spherically symmetric problem, respectively. Sherief and Anwar [68] treated the cylindrically symmetric case.

Another method of inversion for the Laplace transforms in thermoelastic problems is the operational method. Hetnarski [31] used this method successfully to obtain the solution of the Uncoupled problem in the form of series of functions. Sherief and Ezzat [71] obtained the solution of the generalized problem of thermoelasticity in the form of an infinite series of functions by using the same technique, in the context of Lord and Shulman's theory.

The equations of Lord and Shulman's theory for one dimensional problems including heat sources were cast into matrix form by Sherief [70] using the state space and Laplace transform techniques. The resulting formulation is applied to a problem for the whole space with a plane distribution of heat sources. It is also applied to a semispace problem with a traction-free surface and plane distribution of heat sources located inside the medium. The inversion of the Laplace transforms is carried out using a numerical approach. Numerical results for the temperature, displacement and stress distributions are given for both problems.

Nariboli [51] discussed the effect of thermoelastic coupling on the passage of disturbance through an infinite medium with a spherical cavity when its temperature is suddenly changed. The hoop stress at the cavity is seen to reach its steady-state value instantaneously; it then rises to a maximum, falls below the steady-state value and approaches it again. The effect of coupling is seen to reduce this fluctuation of stress throughout and also to reduce the shock character of the disturbance appreciably.

Herrmann [28] extended the variational principles for displacements, stresses and for both displacements and stresses in isothermal elasticity to the coupled processes of thermoelasticity and heat conduction in a three dimensional anisotropic body. It is established that in a stable system the character of these principles concerned with a minimum, maximum and stationary value problem.

Norwood and Warren [58] employed the generalized theory of thermoelasticity to study transient to step time inputs of strain, temperature and stress uniformly distributed over the free surface. The solution is obtained by the use of the Laplace transform with respect to time and the sine transform on space.

Wadhwan [81] considered a spherical cavity in an infinite homogeneous isotropic body, under the action of a general time dependent normal stress and a time dependent temperature, in the context of Lord and Shulman's theory.

Mokrick and Pyrev [48] investigated the analytic properties of solutions of fundamental dynamical problem of the generalized thermoelasticity, based on Lord and Shulman's theory, with finite-rate of heat propagation taken into account. The problem of thermal shock at the surface of a spherical cavity is also studied.

Muller [49,50] obtained such a generalization of coupled thermoelastic theory by analyzing the modified form of entropy production inequality.

Chandrasekharaiah [14] studied one dimensional disturbance in a half-space due to a thermal impulse on the boundary based on Lord and Shulman's and Green and Lindsay's theories.

Chandrasekharaiah [13] has also studied the one dimensional dynamical disturbances in a thermoelastic half-space with plane boundary due to step strain or temperature on the boundary in the context of the Green and Lindsay's theory. The solution is obtained by the use of integral transforms. Short and long time approximations of solutions are deduced and the exact discontinuities in the mechanical and thermal fields are analyzed.

Chandrasekharaiah and Murthy [15] considered the thermoelastic interactions caused in a homogeneous and isotropic unbounded body with a spherical cavity due to a harmonically varying thermal field applied to the stress-free boundary of the cavity.

Furukawa, Noda and Ashida [23,24] studied the generalized theories of thermoelasticity for an infinite body with a cylindrical hole and for an infinite solid cylinder.

Wang and Dhaliwal [83] obtained the general solution of the generalized thermoelastic equations for arbitrary distribution of the body forces and heat sources in an infinite body. They found the short time solutions for the cases of impulsive body force and heat source acting at a point.

Yang, Wang and Chen [84] analyzed the transient response of one-dimensional axisymmetric quasi static coupled thermoelastic problems. Using the Laplace transform with respect to time, the general solutions of the governing equations are obtained in the transform domain.

Bahar and Hetnarski [5] and Anwar and Sherief[4] introduced the state space formulation in coupled and generalized thermoelasticity in the absence of heat sources, respectively.

Misra et al. [45] solved a problem of an infinite solid continuum of an anisotropic, viscoelastic material having a cylindrical cavity in the context of Lord and Shulman's theory. They also [44] gave the solution for the induced temperature and stress fields in an infinite transversely isotropic solid continuum with a cylindrical hole.

Misra, Samanta and Chakrabarti [47] discussed a problem of a half space under the influence of an external primary magnetic field and an elevated temperature field arising out of a ramp-type heating of the surface. They found that the stress distribution and the secondary magnetic field are almost independent of the thermal relaxation time but are significantly dependent on the mechanical relaxation time.

Misra, Chatopadhyay and Samanta [46] studied the generation of stress in a spherically aelotropic homogeneous solid continuum with a spherical cavity.

Misra and Samanta [43] discussed the thermal shock due to sudden heating of the bounding surface of a viscoelastic half-space, they solved the problem with one relaxation time.

Takeuti and Furukawa [79] discussed a thermal shock problem in a plate, they found that it is more important to consider the coupling effects than to consider the inertia effects for thermal shock problems because the coupling effects are much larger than the inertia effects for ordinary metals.

Sternberg and Chakravorty [76] considered the problem of an unbounded thermoelastic body with a spherical cavity whose stress free boundary is subjected to a step in temperature. They solved the problem in the context of the uncoupled theory.

Erbay and Suhubi [22] studied the longitudinal wave propagation in a circular infinite cylinder, they obtained the dispersion relation for the case in which the temperature is kept constant on the surface of the cylinder in the context of Green and Lindsay's theory and Lord and Shulman's theory. Suhubi [78] solved the same problem in Coupled thermoelasticity.

Sherief and Anwar [69] considered the problem of a thick plate subjected to a moving heat source on each face within the context of the theory of generalized thermoelasticity with one relaxation time. They obtained the approximate expressions for the temperature, thermal stresses and displacement.

Chen and Chu [16] analyzed the transient thermal stress distribution of a finite composite hollow cylinder which is heated by a periodically moving line source on its inner boundary and cooled convectively on the exterior surface.

Noda [55,56,57] investigated the transient thermoelastic contact problem in a long circular cylinder, in a short length circular cylinder and in a cylinder with a position dependent heat transfer co-efficient, respectively.

1.2 Discussion of thermoelastic waves:-

The presence of a discontinuity in the material properties generally produces a significant influence on systems of waves propagating through the medium. Consider, for example, the propagation of plane harmonic waves in an unbounded medium consisting of two joined elastic half-spaces of different material properties. In such a composite medium, systems of plane waves can be superposed to present an incident wave in conjunction with reflections and refractions at the interface separating the two media. The wave which emanates from infinite depth in one of the media is called the incident wave. The question then is what combination of additional waves is required in order that the stresses and the displacements are continuous at the interface. These additional waves are called reflected and refracted waves. For the special case of an elastic half-space which adjoins a medium which does not transmit thermal and mechanical waves, the system of waves consists of course of incident and reflected waves only.

Most of the investigations are concerned with plane waves representing disturbances that are uniform in planes of constant phase, i.e., in planes normal to the propagation vector. For bodies with a surface of material discontinuity there are, however, plane waves which are not of constant phase. These waves, which are called surface waves, propagate parallel to the surface of discontinuity. They have the property that the disturbance decays rapidly as the distance from the surface increases. For the free surface the surface waves are known as Rayleigh waves [64], and the surface waves at an interface of two media are called Stonely waves [77]. In the case of the motion of the particles of the medium being parallel to the direction of propagation, the wave is called a Longitudinal wave, and the rotation vanishes, so this type of wave is also often called a dilatational wave, an irrotational wave, a pressure wave, or a P wave (primary, pressure). But in the case of the motion being normal to the direction of propagation, the wave is called a transverse wave, a rotational wave, a shear wave, or an S wave (secondary, shear). When an S wave is polarized, so that all particles of the medium move horizontally during its passage, wave is then termed as an SH wave and; when the particles all move in a vertical plane containing the direction of propagation, the wave is known as an SV wave.

Problems concerning with the propagation of thermoelastic waves in an elastic media have received the attention of many investigators.

Nowacki [60] discussed the propagation of longitudinal waves in an unbounded thermoelastic medium and expressed the temperature and the displacement potential, at any point inside the medium in terms of surface integrals of functions involving temperature and displacement potentials and their normal derivatives.

Lockett [39] investigated the propagation of surface waves in a thermoelastic half space in the context of coupled thermoelastic theory. He studied the effects of thermal properties of the medium on the propagation of Rayleigh waves.

Chadwick and Windle [9] studied the effects of heat conduction upon the propagation of Rayleigh waves in a semi-infinite elastic solid for two special cases:

i) when the surface of the solid is maintained at constant temperature, and

ii) when the surface is thermally insulated.

This problem has again been reconsidered by Chadwick and Atkin [12].

Nariboli and Nayayadhish [52] discussed the one-dimensional wave propagation as an initial and boundary value problem on the basis of coupled thermoelastic equations.

Chow [18] studied the mean waves in a medium with random inhomogeneities within the theory of linear coupled thermo-elasticity. The frequency equation is obtained and analyzed for some special cases. He discussed the problem with random boundary conditions or a randomly varying boundary. Different perturbation methods and two examples are provided.

Chadwick and Seet [10] investigated the thermoelastic wave propagation in a transversely isotropic heat conducting as well as non heat conduction elastic materials. They established that in these types of media three types of elastic waves, namely quasi-longitudinal (QL), quasi-transver (QT), and a purely transver (SH) wave can propagate. Out of these three waves only first two types can be influenced by thermoelastic coupling effects.

Lord and Lopez [41] considered wave propagation in thermoelastic solids at very low temperature when wave type effects are likely to be more prominent.

Wadhwan [82] studied the properties of plane harmonic waves by using the generalized theory of thermoelasticity. It is found that the known results have been modified by introducing the time needed for the acceleration of heat flow.

Jeffreys [35] and Gutenberg [27] considered the reflection of elastic plane waves at a solid half-space. Chadwick and Sneddon [8] studied the propagation of thermoelastic plane waves. Knott [36] derived the general equations for reflection and refraction at plane boundaries.

Sinha and Sinha [72,73] studied the reflection of thermoelastic waves at a solid half-space, and the reflection and refraction of thermoelastic waves at interfaces of two semi-infinite media in contact, with one relaxation time. They also discussed the velocities of the propagation of Rayleigh waves and Stonely waves [74,75].

Nayfeh and Nasser [53] used the equations of Lord and Shulman to study the plane harmonic waves in an unbounded medium.

Agarwal [2,3] extented the Knowles [37] representation theorem for harmonically time dependent free surface waves and plane waves on a homogeneous, isotropic elastic half space in the context of Lord and Shulman's and Green and Lindsay's theories.

Choudhuri [17] studied the effect of rotation and relaxation times on plane waves in the generalized theories of thermoelasticity.

Indeed, if we go to study any problem in elasticity, thermoelasticity or wave propagation in elastic bodies, we must mention some of the important books in these branches like, Love [42], Lekhnitskii [38], Timoshenko and Goddier [80], Parkus [63], Hetnarski [32], Boley and Weiner [7], Nowacki [59,61], Nowinski [62] and Achenbache [1].

1.3 **Basic Equations and Boundary conditions of Thermoelasticity:-**

In a continuous body, the change in the relative position of the points is called a deformation. The stress is the limit of the ratio of traction upon the elementary erea to the area, as this elemental area goes to zero. By the application of suitable forces on a body, both its size and shape can be made to change. When the forces have ceased to act and if the body regains its size and shape, the body is said to be elastic and the property of recovery to an original size and shape is known as elasticity. A body is said to be perfectly elastic if it regains its size and shape completely when the forces producing the deformation are removed.

As far as the elastic properties are concerned, all bodies may be divided, on the one hand, into homogeneous and non-homogeneous, and on the other hand, into isotropic and anisotropic. By a homogeneous body, with regard to its elastic properties, is meant one whose elastic properties are the same at different points. An isotropic body is one in which these properties are the same for all directions drawn through a given point.

The relation between the strain and stress is obtained by Hook's law which states that the components of stress at any point of a body is a linear function of the components of strain at the same point. And the coefficients of this function are called the elastic constants.

1.3.1 Basic Equations:-

We consider a homogeneous, isotropic and thermally conducting elastic medium. We assume that heat sources and external force loading are absent, and take a fixed rectangular cartesian coordinate system (X, Y, Z). In the linearized theory of elasticity, by small deformations we mean that they are small relative to the dimensions of the body. The deformation is described in a very simple manner by the small strain tensor ε, with components

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \qquad (1.3.1)$$

Where u_i are the components of the displacement vector and a comma followed by a subscript denotes partial differentiation with respect to the corresponding coordinate. It is evident that $\varepsilon_{ij}=\varepsilon_{ji}$, i.e. ε is a symmetric tensor of rank two.

The fundamental equations of the classical dynamical coupled theory of thermoelasticity can be written in the form:

o Equations of motion

$$\sigma_{ij,j} + \rho F_i = \rho \ddot{u}_i. \qquad i,j=1,2,3. \qquad (1.3.2)$$

• Heat conduction equation

$$\rho c_E \dot{\theta} + y\, \theta_0\, \dot{u}_{k,k} = k\theta_{,ii}. \tag{1.3.3}$$

• Duhamel-Neumann's relations

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\,(u_{i,j}+u_{j,i}) - y\,\theta\,\delta_{ij}. \tag{1.3.4}$$

where σ_{ij} are the components of the stress tensor, F_i are the components of body forces per unit mass, u_i are the components of the displacement vector, θ is the temperature, θ_0 is the initial temperature, λ, μ are Lame's constants, k is the thermal conductivity, ρ is the density, c_E is the specific heat at constant deformation, α_0 is the co-efficient of linear thermal expansion, δ_{ij} is the kronecker delta and $y=\alpha_0(3\lambda+2\mu)$. A superposed dot denotes differentiation with respect to time.

To distinguish between any problem of thermoelasticity, we observe that, in the presence or absence of the inertia term $(\rho\ddot{u}_i)$ and the Coupled term $(y\,\theta_0\,\dot{u}_{k,k})$, we say that the problem is Dynamic or Quasi-static and Coupled or Uncoupled problem, respectively. Moreover, in the case of the basic equations being independent of time, the problem is said to be Static.

Eliminating σ_{ij} from Equations (1.3.2) and (1.3.4) and rewrite Equation (1.3.3), we find two coupled partial differential equations for temperature and displacement in the form

$$(\lambda+\mu)u_{k,ki} + \mu\nabla^2 u_i - y\,\theta_{,i} = \rho\ddot{u}_i. \tag{1.3.5}$$

$$\rho c_E \dot{\theta} + y\,\theta_0\,\dot{u}_{k,k} = k\theta_{,ii}. \tag{1.3.6}$$

For Lord and Shulman's theory, depending on the modifications in the Fourier's law, we get the following two equations after eliminating σ_{ij}

$$(\lambda+\mu)u_{k,ki}+\mu\nabla^2 u_i-y\ \theta_{,i} = \rho\ddot{u}_i. \qquad (1.3.7)$$

$$\rho c_E(\dot{\theta}+t_0\ddot{\theta})+y\ \theta_0(\dot{u}_{k,k}+t_0\ddot{u}_{k,k})= k\theta_{,ii}, \qquad (1.3.8)$$

where t_0 is the first relaxation time.

However, in Green and Lindsay's theory the basic equations will be

$$(\lambda+\mu)u_{k,ki}+\mu\nabla^2 u_i-y\ (\theta+t_1\dot{\theta})_{,i} = \rho\ddot{u}_i. \qquad (1.3.9)$$

$$\rho c_E(\dot{\theta}+t_0\ddot{\theta})+y\ \theta_0\dot{u}_{k,k} = k\theta_{,ii}, \qquad (1.3.10)$$

where t_1 is the second relaxation time.

So we combine the basic equations for the three different theories of generalized thermoelasticity in the following two partial differential equations for displacement and temperature

$$(\lambda+\mu)u_{k,ki}+\mu\nabla^2 u_i-y\ (\theta+t_1\dot{\theta})_{,i} = \rho\ddot{u}_i. \qquad (1.3.11)$$

$$\rho c_E(\dot{\theta}+t_0\ddot{\theta})+y\ \theta_0(\dot{u}_{k,k}+t_0\delta\ddot{u}_{k,k})= k\theta_{,ii}. \qquad (1.3.12)$$

Here, the use of Kronecker's delta, δ, makes the above fundamental equations possible for the three different theories:-

(1) Classical Dynamical Coupled theory (1956) (C-D)

$t_1 = t_0 = 0, \quad \delta=0.$

(2) Lord and Shulman's theory (1967) (L-S)

$t_1=0, \quad t_0 > 0, \quad \delta=1.$

(3) Green and Lindsay's theory (1972) (G-L)

$t_1 \geq t_0 \geq 0, \qquad \delta = 0.$

In the absence of body forces, the displacement equations of motion follow from Equation (1.3.2) as

$$\mu u_{i,jj} + (\lambda+\mu)\, u_{j,ji} = \rho\, \ddot{u}_i. \tag{1.3.13}$$

In vector notation Equation (1.3.13) can be written as

$$\mu \nabla^2 \mathbf{u} + (\lambda+\mu)\nabla\, \nabla.\mathbf{u} = \rho\, \ddot{\mathbf{u}}. \tag{1.3.14}$$

Let us consider a decomposition of the displacement vector into its potential and rotational functions in the form

$$\mathbf{u} = \text{grad}\ \phi + \text{curl}\ \psi, \tag{1.3.15}$$

substituting into Equation (1.3.14), we obtain the following two equations

$$\nabla^2\phi = \frac{1}{\alpha^2}\, \partial^2\phi/\partial t^2, \qquad \nabla^2\psi = \frac{1}{\beta^2}\, \partial^2\psi/\partial t^2, \tag{1.3.16}$$

where $\alpha = [(\lambda+2\mu)/\rho]^{1/2}$ and $\beta = [\mu/\rho]^{1/2}$ are the longitudinal and transverse wave velocities, respectively.

1.3.2 The Boundary Conditions:-

On the surface S of the undeformed body, the boundary conditions must be prescribed. The following boundary conditions are most common:

(a) *Displacement boundary conditions*: the three components u_i are prescribed on the boundary.

(b) *Traction boundary conditions*: The three traction components t_i

are prescribed on the boundary with unit normal n. Through Cauchy's formula

$$t_i = \sigma_{ij} n_j, \tag{1.3.17}$$

this case actually corresponds to conditions on three components of the stress tensor.

(c) *Displacement* boundary conditions on part S_1 of the boundary and *traction* boundary conditions on the remaining part $S-S_1$.

The initial conditions specify the initial temperature distribution throughout the body. In most problems, the initial temperature is constant. There are five principal boundary conditions which are used in the mathematical theory of heat conduction as idealizations of actual physical processes. Over any portion of the bounding surface of the body, one of the following conditions is usually used:

(1) *Prescribed surface temperature:*

$$\theta(p,t) = f(p,t), \tag{1.3.18}$$

where the point p is on the surface and f(p,t) is a prescribed function.

(2) *Prescribed heat input:*

$$k\ [\partial\theta(p,t)/\partial n] = q(p,t), \tag{1.3.19}$$

where is the outward normal to the surface at the point p.

(3) *Perfectly insulated surface:*

By definition, a perfectly insulated surface is one across which there is no heat flux and thus

$$\partial\theta(p,t)/\partial n = 0. \tag{1.3.20}$$

(4) *Convection boundary conditions:*

In many problems, the heat flux across a bounding surface may be taken as proportional to the difference between the surface temperature $\theta(p,t)$ and the known temperature θ_0 of the surrounding medium. Thus

$$k\ \partial\theta/\partial n = h\ [\theta_0 - \theta(p,t)], \tag{1.3.21}$$

where h is termed as the boundary or film conductance and may vary with space and time in a prescribed manner, h is also called film (or surface) heat transfer co-efficient.

(5) *Two solid bodies in contact:*

If the surface bodies are in perfect thermal contact, their temperature at that surface must be the same. Also, the heat flux leaving one body through the contact surface must be equal to that entering the other body. Thus for a point P on the contact surface

$$\theta_1(p,t) = \theta_2(p,t), \tag{1.3.22}$$

$$k_1\ [\ \partial\theta_1(p,t)/\partial n\ ] = k_2\ [\ \partial\theta_2(p,t)/\partial n\ ], \tag{1.3.23}$$

where subscript 1,2 refer to the two bodies and n is the common normal to the contact surface at p. If there is an imperfect thermal contact between the two bodies, then

$$k_1\ [\ \partial\theta_1(p,t)/\partial n_1] = (1/R)\ [\ \theta_2(p,t) - \theta_1(p,t)], \tag{1.3.24}$$

$$k_1\ [\ \partial\theta_1(p,t)/\partial n_1] = k_2\ [\ \partial\theta_2(p,t)/\partial n_1]. \tag{1.3.25}$$

where R is the contact resistance = 1/h and n_1 is the outward normal referred to body 1, to the contact surface at p.

1.4 An Overview of The Thesis:-

The thesis has been organized into eight Chapters:-

Chapter 1; Introduction.

Chapter 2; Generalized thermoelastic interactions for an infinite body with a spherical cavity.

Chapter 3; Thermal stress distribution for an isotropic solid sphere with two relaxation times.

Chapter 4; Thermal stress in a harmonic field for an infinite body with a circular cylindrical hole.

Chapter 5; Reflection of thermoelastic waves at a solid half-space.

Chapter 6; The propagation of thermoelastic waves at an interface of two semi-infinite media.

Chapter 7; The effect of the second relaxation time on the propagation of Stonely wave.

Chapter 8; Conclusions and scope for the future work.

The main objectives of this study were:-

1- Combining the three different theories in one system of equations to help solve any problem in the generalized thermoelasticity.
2- Comparing the thermal stress distributions in the different theories and discussing the differences due to the presence of the first and second relaxation times.
3- Studying the effect of the second relaxation time in Green and Lindsay's theory on the propagation of thermoelastic waves.

In Chapter II, thermoelastic interactions caused in a homogeneous and isotropic infinite body with a spherical cavity are considered for the two different theories of generalized thermoelasticity, that is, Lord and Shulman's theory and Green and Lindsay's theory. The solution of the problem is obtained in the context of two different cases:-

(a) the boundary of the cavity is constrained and is maintained at a constant temperature.

(b) the boundary of the cavity is stress-free and is maintained at a constant temperature.

By using the Laplace transform technique, the analytical expressions for temperature, displacement and thermal stresses are obtained for small time.

Finally, the numerical values of temperature, displacement and stresses are computed; and the results are presented in the form of graphs to help compare the different theories. From the results we see that:-

- No difference between the two theories (Lord and Shulman's theory and Green and Lindsay's theory) appears in the temperature distribution.
- For the displacement and stresses, there is a difference between the three theories, especially in displacement distribution.
- The temperature increases with the increase of the first relaxation time. However, the change due to the second relaxation time in Green and Lindsay's theory is rather small.
- The effect of an increase in the second relaxation time is very

clear in stress distributions.

- For the constrained body, in the just vicinity of the cavity, the displacement, temperature or stresses are not so much affected by the first relaxation time but at other positions in the medium die out much faster with an increase in the value of the first relaxation time in Lord and Shulman's theory.
- In the second boundary condition, the radial stress diminishes for increasing values of the first relaxation time.

In Chapter III, the distributions of thermal stress and temperature for a homogeneous, isotropic solid sphere are obtained, based on Lord and Shulman's and Green and Lindsay's theories. The solution of the problem is carried out in two different cases:-

(a) the boundary of the sphere is maintained at a constant temperature and the displacement on the surface is constrained.

(b) the boundary of the sphere is maintained at a constant heat flux and the displacement on the surface is constrained.

The problems are analyzed by means of the Laplace transform technique, which on inversion gives the approximate analytical expressions for temperature, displacement and stresses for small time. We calculate the numerical values, and present the results graphically to see the effect of the first and second relaxation times in the two different theories and compare the results with the Classical Dynamical Coupled theory. We observe that:-

- No difference between the two different theories (Lord and Shulman's and Green and Lindsay's theories) appears in the temperature distribution.

- The effect of the second relaxation time is to decrease the radial stress distribution in the just vicinity inside the sphere and increase it to vanish afterwards rather fast. The hoop stress increases with the effect of the second relaxation time in Green and Lindsay's theory.
- The increase of the first relaxation time in Lord and Shulman's theory is to decrease the values of displacement and thermal stresses. But the effect on the temperature field, is to increase it.
- In the second problem, the effect of the first and second relaxation time is clear in the distributions of displacement, temperature and thermal stresses. The difference in the results is very significant compared with the Classical Dynamical Coupled theory due to constant heat flux at the surface of the sphere. This is in agreement with as suggested by Sherief [70].

In Chapter IV, thermoelastic interactions caused in a homogeneous, isotropic infinite body with a circular cylindrical hole due to the effect of harmonic field are discussed. The analytical expressions for temperature, displacement and thermal stresses are obtained in the three different theories in the context of small and large frequency. A numerical example has been given and the results presented graphically to compare the different theories. We find that:-

- In the case of small frequency, there is small effect for the first and second relaxation time in the two different theories of generalized thermoelasticity.

- In the case of large frequency, the temperature distribution in Lord and Shulman's theory coincides with that in Green and Lindsay's theory.

In Chapter V, the effect of the two relaxation times, on the reflection of thermoelastic waves at a homogeneous, isotropic and thermally conducting elastic solid half-space is studied. We deal with two cases of boundary conditions, (i) stress-free surface and (ii) rigid surface, and in each case we discuss the incident wave to be either a P wave or an SV wave. The amplitudes of the reflection co-efficients and the expressions for the partition of energy are obtained in the different cases.

Finally, we find a numerical solution in the case of sand stone, and present the results graphically for the partition of energy with various values of the angle of incidence. From which we conclude that:-

- The partition of energy is independent of the wave number of the incident wave and depend only on the material constants.
- In the absence of thermal coupling the second relaxation time does not affect.
- While the P wave is affected by the presence of the thermal wave, the SV wave remains unaffected.
- The increase of the second relaxation time is to reflect more

energy as P wave and consequently less energy as SV wave.

- The effect of the increase of the first relaxation time is to reduce the effect of the second relaxation time.
- The effect of the second relaxation time is particularly felt at angles of incidence greater than 60^0.

In Chapter VI, the effect of the second relaxation time in Green and Lindsay's theory on the reflection and refraction of thermoelastic waves at an interface of two semi-infinite media in contact is studied. We deal with four cases of interfaces:-

(I) Liquid-Liquid,

(II) Liquid-Solid,

(III) Solid-Liquid,

(IV) Solid-Solid.

In Cases (III) and (IV), we discuss the incident wave to be a P wave or an SV wave. We obtain the amplitudes of the reflection and refraction co-efficients and we find the analytical expressions for the partition of energy. For the numerical results we take the following:

(I) Petroleum-Water,

(II) Water-Sand Stone,

(III) Sand Stone-Water,

(IV) Granite-Sand Stone,

for the four different cases of interfaces, respectively. Finally, the results represented graphically to help compare with that of Lord and Shulman's theory, to see the effect of the second relaxation time. From which we conclude the following important points:

- The partition of energy is independent of the wave number of the incident wave, and depends only on the angle of incidence and the material constants.
- When the first relaxation time increases, the effect of the second relaxation time reduces in the different interfaces.
- In Liquid-Liquid interface, the increase of the first and second relaxation time is to reflect more energy as a P wave, especially for angle of incidence greater than 60^0.
- In Liquid-Solid interface, the increase of the first and second relaxation time is to reflect more energy as a P wave, especially for angle of incidence greater than 30^0. However, the effect in the other two parts of the partition of energy is rather small.
- In Solid-Liquid interface, we conclude that:

  a) In the case of incident P wave, the increase of the first relaxation time is to reflect more energy as P wave and less energy as SV wave, especially at angle of incidence $\geq 50^0$.

  b) In the case of incident SV wave, the increase of the first relaxation time is to reflect less energy as P wave and more energy as SV wave, especially at angle of incidence $\geq 25^0$.
- In Solid-Solid interface, we have two important points:

  a) In the case of incident P wave, the increase of the first relaxation time is to refract less energy as P wave especially at angle of incidence $\geq 40^0$.

  b) But in the case of incident SV wave, the increase of the first relaxation time is to refract less energy as SV wave for angle of incidence $I \geq 25^{0}$.

In Chapter VII, the effect of the second relaxation time in Green and Lindsay's theory on the velocity of Stonely wave is discussed. The characteristic equation for the velocity of Stonely wave in the three different theories has been given. We also calculate the variation of the density ratios for different media.

In Chapter VIII, the important results and observations from this thesis are concluded. The general conclusions are divided into two parts:-

1) The effect of the first and second relaxation time on a boundary value problems.
2) The effect of the first and second relaxation time on the propagation of thermoelastic waves.

In the last of that chapter, scope of the future work is given.