Abstract

Two points are *visible* if the line segment joining them is not obstructed by any input object. A generalization of visibility, called *link visibility*, has been studied extensively, where two points inside a polygon are called *link-j* visible if they can be joined by a polygonal path (called a *link path*) of j or fewer line segments that are subsets of the input polygon. The link path between two points may consist of several left and right turns. We introduce two new generalizations of visibility viz., *visibility with reflection* and *convex visibility*, by restricting the notion of link visibility in two different ways in order to model two different applications that cannot be modelled using link visibility.

Two points are *visible after k reflections*, if there is a link path of $k+1$ line segments joining them that bends only at the polygon boundary. If each bend follows the standard laws of reflection we call it *specular reflection*. Otherwise, if each bend is only required to be inside the polygon, we call it *diffuse reflection*. This is meaningful in illumination modelling applications where boundary walls reflect light. Two points are *convex visible*, if the Euclidean shortest path joining them makes only left turns or only right turns. If two points p and q are convex visible, the shortest path joining them is a link path that may be visualized as a jointed robot arm, making it possible to reach a job at q from a robot placed at p, using only right turns or only left turns in the arm. This model is chosen to alleviate the inconvenience of using zig-zag robot arms. Specific results of this thesis on visibility with reflection and convex visibility are stated below.

We establish bounds on the combinatorial complexity of the regions visible, inside a simple n-gon, due to multiple specular and diffuse reflections, and design algorithms for computing these visible regions. It is shown that the region visible due to specular or diffuse reflections may have $\Omega(n^2)$ edges; this bound is tight for single reflection. In case of specular reflections, the complexity of the visible region may increase exponentially with the number of reflections considered. Specifically, the visible region due to at most $k < n/c$, where $c > 1$ is a constant, specular reflections may have $\Omega((n/k)^{2k})$ boundary components (holes). We show that this lower bound is tight with a matching upper

bound for fixed $k < n$.

In contrast to single specular reflection, the visible region under single diffuse reflection is simply connected and thus has a single boundary component. For multiple diffuse reflections, we show that the complexity of the visible region due to at most k reflections is $O(n^{2\lceil \frac{k+1}{2} \rceil + 1})$, and therefore, provably smaller than the complexity of the visible region under $k > 2$ specular reflections in the worst case. For $k \geq \lceil (n-3)/2 \rceil$, it is established that the entire input polygon becomes visible after k diffuse reflections, making the visible region simply connected and identical to the given polygon.

Linear time algorithms are proposed to compute the regions (i) convex visible from a given point and (ii) *weakly convex visible* from a given line segment inside the polygon. An $O(n^2)$ algorithm is presented for determining a line segment (if any) so that every point in the given n-vertex polygon is convex visible from some point on the line segment.