

## SYNOPSIS

The thesis concerns with the study of blade vibration problem. The problem is stated and the previous work including that carried out by the author and his predecessors, in the laboratory where the author is working, is reviewed.

Extensive experiments have been conducted to determine the effect of taper on the natural frequencies, in bending and torsion. Thirty one pairs of blades with different tapers having rectangular cross-section, have been tested and their nodal patterns drawn. The experimental technique is outlined. In determination of nodal patterns two different properties were used.

- (1) Phase difference
- and
- (2) Effect of damping.

An equation has been derived for the fundamental bending mode of a blade, taking into account the effect of taper, by Galerkin's method. These results are compared with the correction factors derived by Martin, and with the computed results of Housner and Keightly. No attempt has been made to study theoretically the effect of shear deflections and rotary inertia on the tapered blades in this work. However the correction factors available for uniform beams for these effects have been applied for all the blades, so as to compare the results with the experimental values.

For determination of the uncoupled torsional frequencies, an equation has been obtained for the fundamental mode by the collocation method. The tabular procedure used by Priests for free-free systems has been extended for fixed-free systems by means of which a frequency equation could be set up, to determine the natural frequencies. The familiar Holzer's procedure is used to determine the fundamental torsional frequencies of some of the blades. To determine the effect of taper on the first three modes in torsion, the correction factors were calculated by the Holzer's numerical procedure with the help of IBM 1620 digital computer.

The uncoupled natural frequencies obtained by the theoretical methods are compared with the corresponding experimental values. Experimental results obtained for some of the coupled modes have also been reported.

The thesis consists of the following chapters.

Chapter 1. Introduction

2. Literature survey
3. Theoretical analysis on bending vibrations of cantilever beams
4. Theoretical analysis torsional vibrations of cantilever beams
5. Calculations of frequencies of the blades under investigation
6. Experimental set up and observation
7. Discussion of the results
8. Conclusions
9. Scope of future work.

List of Symbols

$E$	=	Youngs modulus (lb/in <sup>2</sup> )
$I$	=	moment of inertia of cross-section (in <sup>4</sup> )
$y$	=	bending deflection (in)
$m$	=	mass per unit length (lb.Sec <sup>2</sup> /in)
$\omega$	=	angular frequency (rad/sec)
$x$	=	distance along the length of the beam from fixed end (in)
$L$	=	length of the beam (in)
$X$	=	$\frac{x}{L}$ (non-dimensional)
$b$	=	breadth of the beam (in)
$t$	=	thickness of the beam (in)
$d$ & $\beta$	=	breadth and thickness parameter (non-dimensional)
$A$	=	area of cross section (in <sup>2</sup> )
$d$	=	radius of gyration = $\sqrt{\frac{I}{A}}$ (in)
$m_0, I_0$ & $A_0$	=	values corresponding to the root section
$\lambda$	=	$\frac{m_0 \omega^2}{EI_0}$ (equation 3.5)
$y', y''$ & $y'''$	=	1st, 2nd and 3rd differentials of $y$ w.r.t. $x$

$\epsilon$	= error in the differential equation
$\rho$	= density of the material (lb/in <sup>3</sup> )
$\Omega$	= non-dimensional frequency
H & D	= Parameters as defined by Housner explained in Sec.3.4
$\gamma_1, \gamma_2 \& \gamma_3$	= Values of non-dimensional frequencies as explained in Sec.3.4
C	= Torsional rigidity $\frac{\text{lb in}^2}{\text{rad}}$
K	= numerical factor defined in equation 4.1
G	= Modulus of rigidity 12.0 x 10 <sup>6</sup> Psi.
c, m, r	= constants defined in equation 4.2
$\theta$	= torsional deflection (rad)
g	= acceleration due to gravity (in/sec <sup>2</sup> )
$I_p$	= Polar moment of inertia of (in <sup>4</sup> ) cross-section
$A_0, A_1 \& A_2$	= constants defined in equation 4.13
$q_i$	= torsional stiffness of ith shaft in Fig.7, $\frac{\text{lb in}}{\text{rad}}$
$J_i$	= Mass moment inertia of ith rotor in Fig.7 (lb. in. Sec <sup>2</sup> )

$\theta_i$  = deflection of  $i$ th rotor in torsion

$\dot{\theta}_i, \ddot{\theta}_i$  = differentials of  $\theta_i$  w.r.t. time

$m_i$  = torsional moment of  $i$ th shaft as given in equation 4.30

$Z$  =  $\omega^2$

~~$A_{i1}, A_{i2}$~~   $\theta_i, M_i$  = amplitudes of  $\theta_i$  and  $m_i$

$A_{i1}, A_{i2}, \dots, A_{ii}$  = coefficients as defined in equation 4.38

$B_{i+1,1}, B_{i+1,2}, \dots, B_{i+1,i}$  = coefficients as defined in equation 4.38

$J'_i, q'_i$  = non dimensional values of  $J$  and  $q$

$J^*, q^*$  = reference values of  $J$  and  $q$

$\eta$  =  $Z \frac{J^*}{q^*}$

$S$  = shear distribution factor

$F_b$  = correction factor for taper on bending vibration

$F_t$  = correction factor for taper on torsional vibration

$F_s$  = correction factor for shear on bending vibration

$F_r$  = correction factor for rotary inertia on bending vibration

- $\omega_b$  = corrected frequency in bending for the effect of taper  
 $\omega_{bs}$  = corrected frequency in bending for the effects of taper and shear  
 $\omega_{brs}$  = corrected frequency in bending for the effects of taper shear and rotary inertia  
 $p_n$  = A number defined in equation 3.3  
 $m_i$  = coefficients for 1st, 11nd.. modes in bending as given in equation 3.4  
 $a_0, a_1 \dots a_4$  = constants defined in equation 3.8  
 $S_1, S_2 \dots S_6$  = certain integrals given in equation 3.17  
 $\underline{a_1}, \underline{a_2} \dots \underline{a_5}$  = constants in equation 3.24  
 $r_1, r_2 \dots r_5$  = roots of the frequency equation 4.46b