Chapter I

INTRODUCTION

1.1 Hydrodynamic Lubrication

Lubrication is the art of reducing frictional resistance between two surfaces in relative motion by means of some kind of substance between the surfaces. Such a substance is called a lubricant. The function of the lubricant is to hold the moving surfaces apart allowing them to slide on each other with minimum effort. Lubrication is indispensable in bearings which support moving members in a machine.

Though the use of lubricants to reduce wear and resistance was known to mankind for quite a long time, it was not a subject of any theoretical analysis till the end of nineteenth century. Petrov (1883) was apparently the first to make significant attempt to analyse theoretically the friction effect of fluid film lubrication. However, the systematic development of the theory of hydrodynamic lubrication stems directly from the experiments conducted by Tower (1883) and their interpretation by Reynolds (1886). In a classical paper Reynolds obtained a differential equation, which bears his name now, making use of hydrodynamical laws and showed that the motion of a viscous fluid in narrow clearance of bearing surfaces develop such high pressure as to sustain a load. Kingsbury (1897) verified the hydrodynamic theory of lubrication by experiments on air-lubricated bearings.

Later on, many people contributed to the advancement of fluid film lubrication. Mention may be made of Sommerfeld (1904) who provided elegant theoretical extensions to the journal bearing problems, Michell (1905) who extended Reynolds theory to include side leakage, Rayleigh (1918) who found that the optimum profile for the maximum load capacity of a slider bearing was a step function, Kingsbury (1931) who used electrical analogy method to obtain the solution of a bearing problems with side leakage and Christopherson (1942) who used numerical methods to solve bearing problems.

In recent years numerous engineers and applied mathematicians have contributed to the theory of hydrodynamic lubrication. As a result, a huge amount of literature now exists in the form of books by Shaw and Macks (1949), Michell (1950), Fuller (1956), Pinkus and Sternlicht (1961), Tipei (1962), Gross (1962) and Cameron (1966, 1971).

1.2 Hydromagnetic Lubrication

Magnetohydrodynamics deals with the motion of electrically conducting fluids in a magnetic field. The motion of electrically conducting fluids across a magnetic field induces an electric field which gives rise to an electric current. The electric current interacts with the magnetic field to produce an electromagnetic body force which changes the flow field.

The use of the concepts of magnetofluid dynamics to lubrication has been receiving attention for the last fifteen years. Liquid metals like sodium, mercury are used as lubricants in some high temperature applications. These fluids have several advantages over ordinary lubricating oils, such as high thermal conductivity and ability to withstand high temperatures. But their lubricating property is poor due to low viscosity. Consequently, the load capacity is reduced. However, the load capacity may be increased by the electromagnetic effects. The hydromagnetic bearing can support a load when a similar hydrodynamic bearing has no load capacity. Under certain circumstances, the frictional force can also be reduced by the electromagnetic effects.

During the last fifteen years more than a hundred articles have been published. We mention here some of the earliest papers on hydromagnetic lubrication.

Snyder (1962) and Fucks and Uhlenbusch (1962) investigated inclined slider bearings with transverse magnetic field and found that the load capacity increased due to the magnetic field. Rayleigh problem of optimum slider was extended to hydromagnetic lubrication by Osterle and

Young (1962). Elco and Hughes (1962) studied inclined slider bearing with a tangential magnetic field.

The first analysis of hydromagnetic externally pressurized bearing (thrust bearing) was due to Hughes and Elco (1962). The field configurations considered by them were an axial magnetic field with radial current and a radial magnetic field with an axial current. It was found that the load capacity depended on electromagnetic interactions and the torque could be made zero by supplying current to the fluid from an external source. Elco and Hughes (1962) also investigated thrust bearing with an axial current induced pinch.

The hydromagnetic squeeze films were investigated by Kuzma (1964), Kuzma, Maki and Donelly (1964). The latter authors conducted the first experiment on hydromagnetic lubrication. The experiment was on squeeze film using mercury between two circular plates with a transverse magnetic field. The experiment proved that mercury is a better lubricant than an oil of the same viscosity when the magnetic field is applied. Electromagnetic effects is to retard the squeeze and increase the time of approach.

1.3 Polar Fluid Theory

Classical continuum mechanics is based on the assumption that all the material bodies possess continuous mass density, the laws of motion and axioms of constitution are valid for every part of the body regardless of its size. To treat the flow of fluids with microstructure like those containing some additives, suspensions are granular matter, the classical theory is inadequate and one has to look for a generalization of the classical concepts.

During the last thirteen years, various theories of microcontinuum have been developed. Eringen (1964) introduced the theory of simple microfluids which exhibit certain microscopic effects arising from local deformation and microrotation of substructure. As a subclass of microfluids, Eringen (1966) introduced micropolarfluids in which the local fluid elements are allowed to undergo only rigid rotation without strech. Physically micropolarfluids may represents fluids with rigid spherical substructure.

The basic equations of motions of fluid with rigid spherical substructure are also obtained by various workers from different view points. Mention may be made of Aero, Bulygin and Kuvshinskii (1965), Allen and Kline(1968), Cowin (1968) and Erdogan (1972). Various names like asymmetric hydrodynamics, polar fluid theory, have been used to describe micropolar fluid theory.

1.4 Basic Equations of Polar Fluid Theory

In this section we record the balance equations of mass, linear momentum, angular momentum and energy.

If F is any function of the position and time, the Reynolds transport theorem states that

$$\left(\int_V F \, dV \right)^{\bullet} = \int_V (\dot{F} + F \, v_{k,k}) \, dV \qquad (1.1)$$

where F is a scalar or a vector or a tensor of any order, the superposed dot denotes the material time derivative, V the material volume, v_i the velocity vector, an index followed by a comma denotes partial differentiation and a repeated index denotes summation over that index from 1 to 3.

The mass conservation requires that

$$\left(\int_V \rho \, dV \right)^{\bullet} = 0$$

or by equation (1.1) we have

$$\dot{\rho} + \rho \, v_{k,k} = 0 \qquad (1.2)$$

where ρ is the density of the fluid. The linear momentum principle is

$$\left(\int_V \rho \, v_i \, dV \right)^{\bullet} = \int_{\partial V} T_{ji} \, n_j \, dS + \int_V \rho \, b_i \, dV$$

and by (1.1) and (1.2) we obtain

$$\rho \dot{v}_i = T_{ji,j} + \rho b_i \tag{1.3}$$

where T_{ij} is the stress tensor and b_i is the body force per unit mass.

The angular momentum principle is

$$\left[\int_V \rho \, (J_i + e_{ijk} \, x_j \, v_k) dV \right]^{\circ}$$

$$= \int_{\partial V} (e_{ijk} \, x_j \, T_{pk} + M_{pi}) \, n_p \, dS$$

$$+ \int_V \rho (g_i + e_{ijk} \, x_j \, b_k) dV$$

Here J_i is the internal angular momentum, e_{ijk} denotes the alternating tensor, M_{ij} is the couple stress tensor and g_i is the body couple per unit mass. Using (1.1), (1.2) and (1.3) we have

$$\rho \dot{J}_i = e_{ijk} \, T_{jk} + M_{ji,j} + \rho \, g_i \tag{1.4}$$

The substructure is assumed to be rigid so that the spin (internal) angular momentum may be written as

$$J_i = I_{ik} \nu_k$$

where I_{ik} is the moment of inertia, per unit mass and ν_k is the angular velocity of rotation of the element. If the fluid is assumed to be isotropic, we have

$$I_{ik} = J \delta_{ik}$$

where J is a constant and δ_{ik} is Kronecker's delta. Hence (1.4) takes the form

$$\rho J \dot{\nu}_i = e_{ijk} T_{jk} + M_{ji,j} + \rho g_i \qquad (1.5)$$

the balance of energy is given by

$$\left[\int_V \frac{\rho}{2} (v_k v_k + J \nu_k \nu_k + 2e) dV \right]^{\cdot}$$

$$= \int_{\partial V} (v_i T_{ji} + \nu_i M_{ji} - q_j) n_j \, dS$$

$$+ \int_V \rho (v_i b_i + \nu_i g_i + Q) \, dV$$

where e is the internal energy per unit mass, q_i is the heat flux vector, Q is the heat source per unit mass. Multiplying (1.4) by v_i and (1.5) by ν_i we get

$$\rho\dot{e} = (v_{i,j} + e_{ijk}\ \nu_k)\ T_{ji} + \nu_{i,j}\ M_{ji} - q_{i,i} + \rho Q. \tag{1.6}$$

The principle of entropy states the Clausius-Duhem inequality

$$\left(\int_V \rho s\ dV\right)^{\bullet} - \int_V \frac{\rho Q}{\Theta}\ dV + \int_{\partial V} \frac{q_i n_i}{\Theta}\ dS \geqslant 0$$

or, in the differential form,

$$\rho\Theta\dot{s} - \rho Q + q_{i,i} - \frac{q_i\ \Theta_{,i}}{\Theta} \geqslant 0 \tag{1.7}$$

Elimination of Q from (1.6) and (1.7) yields

$$\rho(\Theta\dot{s} - \dot{e}) - \frac{q_i\ \Theta_{,i}}{\Theta} + (v_{i,j} + e_{ijk}\ \nu_k)T_{ji} + \nu_{i,j}\ M_{ji} \geqslant 0. \tag{1.8}$$

Introducing Helmholtz free energy per unit mass, $\Psi = e - \Theta s$, the inequality (1.8) takes the form

$$-\rho(\dot{\Psi} + s\ \dot{\Theta}) - \frac{q_i\ \Theta_{,i}}{\Theta} + (v_{i,j} + e_{ijk}\ \nu_k)\ T_{ji} + \nu_{i,j}\ M_{ji} \geqslant 0 \tag{1.9}$$

The inequality (1.9) imposes certain restrictions on the material constants of the fluid appearing in the constitutive equations.

Constitutive Equations for Polar Fluids

The constitutive equations for stress tensor and couple stress tensor as introduced by Eringen (1966) are

$$T_{ij} = (-p + \lambda v_{k,k})\,\delta_{ij} + \mu(v_{i,j} + v_{j,i})$$

$$+ \frac{k}{2}\,(v_{j,i} - v_{i,j} - 2e_{ijk}\,\nu_k) \qquad (1.10)$$

$$M_{ij} = \alpha\,\nu_{k,k}\,\delta_{ij} + \beta\,\nu_{i,j} + \gamma\nu_{j,i} \qquad (1.11)$$

where λ, μ are the classical viscosity coefficients, α, β, γ are the new material constants peculiar to the polar fluid and p is the pressure.

The inequality (1.9) restricts the constants

$$3\lambda + 2\mu \geqslant 0 \quad \mu \geqslant 0 \quad k \geqslant 0$$

$$3\alpha + \beta + \gamma \geqslant 0 \quad \gamma \geqslant 0 \quad -\gamma \leqslant \beta \leqslant \gamma \qquad (1.12)$$

<u>Field Equations</u>

We now collect here the field equations of polar fluid dynamics

$$\dot{\rho} + \rho v_{k,k} = 0 \tag{1.13}$$

$$\rho \dot{v}_i = - p_{,i} + \rho b_i + (\lambda + \mu - \frac{k}{2}) v_{k,ik}$$

$$+ (\mu + \frac{k}{2}) v_{i,kk} + k\, e_{ijk}\, \nu_{k,j} \tag{1.14}$$

$$\rho J\, \dot{\nu}_i = \rho g_i + (\alpha + \beta)\, \nu_{k,ik} + \gamma\, \nu_{i,kk}$$

$$+ k(e_{ijk}\, v_{k,j} - 2 \nu_i) \tag{1.15}$$

Equation (1.13) is in fact (1.2) and (1.14), (1.15) are obtained from (1.3), (1.5) by using (1.10) and (1.11).

In polar fluid dynamics the 7 unknown functions ρ, v_i, ν_i are to be obtained by solving the 7 equations (1.13), (1.14) and (1.15). For incompressible flows the seven unknowns p, v_i, ν_i are to be determined by solving the 7 equations

$$v_{k,k} = 0 \qquad (1.16)$$

$$\rho \overset{\circ}{v}_i = -p_{,i} + \rho b_i + (\mu + \frac{k}{2})\, v_{i,kk} + k e_{ijk}\, \nu_{k,j} \qquad (1.17)$$

and (1.15).

1.5 Couple Stress Theory of Fluids

The couple stress theory of fluids advanced by Stokes (1966) assumes that the internal angular momentum vanishes and the microrotation equals half the vorticity, that is,

$$J = 0 \qquad (1.18)$$

$$\underset{\sim}{\nu} = \underset{\sim}{\omega} = \frac{1}{2}\nabla \times \underset{\sim}{v} \qquad (1.19)$$

The symmetric part of stress tensor and the couple stress tensor are given by

$$T_{(ij)} = (-p + \lambda v_{k,k})\, \delta_{ij} + \mu(v_{i,j} + v_{j,i}) \qquad (1.20)$$

$$M_{ij} = \beta\omega_{i,j} + \gamma\omega_{j,i} \qquad (1.21)$$

The antisymmetric part of stress tensor $T_{[ij]}$ is obtained from (1.5) on multiplication with e_{ipq} and the use of the identity

$$e_{ijk}\, e_{ipq} = \delta_{jp}\, \delta_{kq} - \delta_{jq}\, \delta_{kp}$$

Thus

$$T_{[ij]} = -\frac{1}{2}\, e_{ijk}\, (\rho g_k + M_{pk,k}) \qquad (1.22)$$

Substituting for M_{pk} from (1.21) in (1.22) we get

$$T_{[ij]} = -\frac{\rho}{2}\, e_{ijk}\, g_k + \frac{\gamma}{4}\, \nabla^2 (v_{i,j} - v_{j,i}) \qquad (1.23)$$

$$M_{ij} = \frac{\beta}{2}\, e_{ipq}\, v_{q,pj} + \frac{\gamma}{2}\, e_{jkq}\, v_{q,ki} \qquad (1.24)$$

Adding (1.20) and (1.23) we get

$$T_{ij} = (-p + \lambda v_{k,k})\, \delta_{ij} + \mu (v_{i,j} + v_{j,i})$$

$$+ \frac{\gamma}{4}\, \nabla^2 (v_{i,j} - v_{j,i}) - \frac{\rho g_k}{2}\, e_{ijk} \qquad (1.25)$$

Now using (1.25) the field equations are

$$\dot{\rho} + \rho v_{k,k} = 0 \qquad (1.26)$$

$$\rho \dot{v}_i = \rho b_i + \frac{1}{2} e_{ijk} (\rho g_k)_{,j} - p_{,i} + (\lambda + \mu + \frac{\gamma}{4}\, \nabla^2) v_{j,ji}$$

$$+ (\mu - \frac{\gamma}{4}\, \nabla^2)\, v_{i,jj} \qquad (1.27)$$

These are four equations for the four unknowns ρ, v_i. For incompressible flow the four unknowns p, v_i are obtained by solving the four equations

$$v_{k,k} = 0 \tag{1.28}$$

$$\rho\dot{v}_i = \rho b_i + \frac{\rho}{2} e_{ijk} g_{k,j} - p_{,i} + (\mu - \frac{\gamma}{4} \nabla^2) v_{i,jj} \tag{1.29}$$

It is interesting to note from (1.21) that M_{ij} is traceless, i.e., $M_{kk} = 0$. The addition of a spherical tensor $m\delta_{ij}$ to M_{ij} results into an addition of $(1/2)e_{ijk}\, m_{,k}$ to $T_{[ij]}$. However the field equations are unaffected since $e_{ijk}\, m_{,kj} = 0$. Thus the trace of the couple stress tensor and the antisymmetric part of stress tensor are indeterminate. Hence this theory is some times called indeterminate couple stress theory.

In a series of papers, Stokes (1966, 1968, 1971) solved a number of boundary value problems considering equations (1.28) and (1.29)

1.6 Derivation of Couple Stress Theory from Polar Fluid Theory

The couple stress theory of fluids is a particular case of polar fluid theory. To derive the couple stress theory from polar fluid theory we assume that

$$J = 0 \quad (1.30)$$

$$k \longrightarrow \infty \quad (1.31)$$

Let us rewrite the field equations of the polar fluid (1.13), (1.14) and (1.15) in the index free notation :

$$\dot{\rho} + \rho \nabla . \underset{\sim}{v} = 0 \quad (1.32)$$

$$\rho \overset{\circ}{\underset{\sim}{v}} = -\nabla p + \rho \underset{\sim}{b} + (\lambda + 2\mu) \nabla (\nabla . \underset{\sim}{v}) - \mu \nabla \times \nabla \times \underset{\sim}{v}$$

$$+ \frac{k}{2} \nabla \times (2 \underset{\sim}{\nu} - \nabla \times \underset{\sim}{v}) \quad (1.33)$$

$$0 = \rho \underset{\sim}{g} + (\alpha + \beta + \gamma) \nabla (\nabla . \underset{\sim}{\nu}) - \gamma \nabla \times \nabla \times \underset{\sim}{\nu}$$

$$+ k (\nabla \times \underset{\sim}{v} - 2 \underset{\sim}{\nu}) \quad (1.34)$$

Dividing (1.34) by k and taking the limit as $k \rightarrow \infty$, we get

$$\nabla \times \underset{\sim}{v} = 2 \underset{\sim}{\nu} \quad (1.35)$$

showing that the microrotation is constrained as half the vorticity vector.

We note that though $k \rightarrow \infty$ and $\nabla \times \underset{\sim}{v} \longrightarrow 2 \underset{\sim}{\nu}$ the product is finite, indeed,

$$k(2\underset{\sim}{\nu} - \nabla \times \underset{\sim}{v}) = \rho \underset{\sim}{g} - \frac{\gamma}{2} \nabla \times \nabla \times \nabla \times \underset{\sim}{v} \qquad (1.36)$$

On using (1.36) in (1.33) we get

$$\rho \dot{\underset{\sim}{v}} = -\nabla p + \rho \underset{\sim}{b} + (\lambda + 2\mu) \nabla (\nabla . \underset{\sim}{v}) - \mu \nabla \times \nabla \times \underset{\sim}{v}$$

$$+ \frac{1}{2} \nabla \times (\rho \underset{\sim}{g}) - \frac{\gamma}{4} \nabla \times \nabla \times \nabla \times \nabla \times \underset{\sim}{v} \qquad (1.37)$$

or

$$\rho \dot{\underset{\sim}{v}} = -\nabla p + \rho \underset{\sim}{b} + \frac{1}{2} \nabla \times (\rho \underset{\sim}{g}) + (\lambda + \mu + \frac{\gamma}{4} \nabla^2) \nabla (\nabla . \underset{\sim}{v})$$

$$+ (\mu - \frac{\gamma}{4} \nabla^2) \nabla^2 \underset{\sim}{v} \qquad (1.38)$$

which is the same as (1.27) the field equations of the couple stress fluid.

To derive the constitutive equations (1.24) and (1.25) from polar fluid theory, we note that (1.24) follows immediately from (1.11) through (1.35). Now (1.10) can be written as

$$T_{ij} = (-p + \lambda v_{k,k}) \delta_{ij} + \mu(v_{i,j} + v_{j,i})$$

$$+ \frac{k}{2} e_{ijk} (\nabla \times \underset{\sim}{v} - 2\underset{\sim}{\nu})_k \qquad (1.39)$$

Use of (1.36) in (1.39) gives

$$T_{ij} = (-p + \lambda v_{k,k})\, \delta_{ij} + \mu(v_{i,j} + v_{j,i})$$

$$- \frac{e_{ijk}}{2} \left[\rho g_k + \frac{\gamma}{2}\, e_{k\alpha\beta}\; v_{\beta,\alpha pp} \right] \qquad (1.40)$$

which is the same as (1.25)

1.7 Present Investigation

The remaining six chapters deal with certain theoretical investigations of lubrication problems. Chapter II to IV are devoted to hydromagnetic lubrication and chapters V to VII to the study of the performance of bearings lubricated by fluids with couple stress.

The study in Chapter II of a viscous incompressible electrically conducting fluid between two parallel circular plates one of which is oscillating axially in the presence of a transverse magnetic field shows that the pressure in the fluid film increases both with Hartmann number and Reynolds number. The increase in pressure due to inertia is not significant for large Hartmann numbers. The effects of inertia and magnetic field on the pressure distribution when the plate is in its downward motion is qualitatively similar to those effects in a hydromagnetic squeeze film bearing.

A simple electromagnetic device for increasing the efficiency of a hydromagnetic squeeze film bearing between rectangular plates is considered in Chapter III. The lubricant is viscous incompressible electrically conducting and the magnetic field is obtained by a line current. It is shown that the squeeze is slowed down by application of magnetic field.

Almost all the theoretical investigations in hydromagnetic lubrication concern with incompressible lubricants. Due to the nonlinearity of the Reynolds equation in magnetogasdynamic lubrication analytical solutions are rarely possible and one has to resort to numerical solutions. In Chapter IV we consider a magnetogasdynamic thrust bearing with an azimuthal magnetic field and an axial current. It is shown that the load supporting capacity can be increased by such a field configuration and a load can be sustained even without the flow of the lubricant.

Chapter V is devoted for investigation of slider bearings lubricated by incompressible fluids with couple stress. The modified Reynolds equation is derived and the expressions for flow rate, load capacity, centre of pressure, frictional force and the friction coefficient are obtained for a general film thickness. Specialising the film thickness we have obtained detailed solution for inclined slider, step slider and composite slider. Numerical results have been obtained for the couple stress parameter L = 0, 0.01, 0.1, 0.5, 1 and 10. It is found that the load capacity and frictional force increase with L, however the more important parameter in lubrication, namely the frictional coefficient decreases as the couple stress parameter increases.

The slider profile which gives the maximum load capacity of a slider bearing lubricated by fluid with couple stress is considered in Chapter VI. It is shown that the optimum profile is a step function with riser location and step hight ratio depending on the couple stress parameter. The results reduce to those of the classical Rayleigh problem when the couple stress parameter vanishes. The optimum load capacity increases with couple stress parameter.

Chapter VII deals with squeeze film between rectangular plates, between circular plates and a thrust bearing. It is shown that the time for a given fall is an increasing function of couple stress parameter. From the investigation of thrust bearings, it is shown that to support a given load less pump work is needed if the lubricant is a fluid with couple stress and for a given flowrate, more load can be supported.