CHAPTER - 0

INTRODUCTION

0.1 MOTIVATION OF THE WORK UNDER INVESTIGATION

Wald's (1947) Sequential Probability Ratio Test (SPRT) procedure for testing one simple statistical hypothesis against another has an optimum property that for preassigned error probabilities of the first and the second kind, there is no other test procedure that requires on the average a smaller number of observations to take a decision (to either reject or accept the null hypothesis) when either of the hypotheses is true, than what is required by the SPRT procedure. However, Wald's SPRT procedure doesnot in general minimize the least upper bound of the number of observations required on the average to make a decision for an arbitrary value of the parameter concerned. In fact, for the problem of testing a simple hypothesis $\mathcal{H}_0 : \mu = \mu_0$ against a simple alternative $\mathcal{H}_1 : \mu = \mu_1$, μ being the concerned parameter, an uniformly most efficient SPRT may actually be less efficient than the optimum fixed sample test when the true value of μ is in the neighborhood of $\bar{\mu} = (\mu_0 + \mu_1)/2$ [cf. Ghosh (1970), p-141].

In order to reduce the least upper bound of the average of the sample number (ASN) required to make a terminating decision, several modifications of the SPRT have been suggested, among others, by Anderson (1960) and Read (1971). Among these modifications of the SPRT, Anderson's modified SPRT procedure reduces the maximum ASN at least for the problem of testing a normal mean with known variance, more than any other procedure. However a question may be raised, as has been done by Weiss (1962) while discussing his Bayesian truncated sequential test procedure, as to whether Anderson's procedure gives the best possible result as far as the reduction of the maximum ASN is concerned.

In this thesis we consider certain modifications of Anderson's and Read's procedures which have the property of reducing the maximum ASN of Wald's SPRT and show that it is possible to find modifications, of the SPRT procedure, which reduce the maximum ASN even more than Anderson's procedure. It is observed that an attainable lower bound of the maximum ASN can be made quite close to Hoeffding's (1960) theoretical lower bound.

0.2 AN OVERVIEW OF THE THESIS

This thesis, containing eight chapters, is divided into three parts :

Part-I :- Review of Previous Work ;

Part-II:- Some Modifications of the SPRT : Testing the Normal Mean with Known Variance ;

Part-III:- Some Modifications of the SPRT : Testing the Binomial Proportion.

Part-I, consisting of two chapters, gives a brief review of the previous work on various topics of the SPRT in general and on the problem of reduction of the maximum ASN in particular. It also includes an uptodate bibliography, classified into fourteen topics, on the SPRT. Part-II, consisting of five chapters and Part-III, consisting of one chapter deal with certain modifications of the SPRT to reduce the sample size respectively for the problem of testing the normal mean with known variance and the problem of testing the binomial proportion.

Chapter-1 gives a brief introduction to the origin of the SPRT, its growth and later developments. The earlier work on the problem under investigation, viz., the problem of reducing the ASN of the SPRT, has been discussed in Chapter-2.

An upto date bibliography on the SPRT, with subject classifications, connected to the discussions made in Chapters 1 and 2, has been compiled and is given as an appendix of Part-I of the thesis. A separate list of references, given at the end of Part-III of the thesis, is connected to the discussions of the chapters other than Chapters - 1 and 2.

The first chapter of Part-II, Chapter-3, deals with a modification of Read's (1971) partial SPRT (or the PSPRT). The PSPRT consists in drawing an initial fixed sample prior to an SPRT. Read has shown numerically that at least for the problem of testing a normal mean with known variance, his PSPRT has the property of reducing Wald's maximum ASN. The proposed modification of the PSPRT (MPSPRT) consists in drawing initially two fixed-samples, the second one being conditional on the first, prior to an Wald-SPRT. The Operating Characteristic (OC) and the average sample number (ASN)functions of the MPSPRT for the problem of testing a Koopman - Darmois parameter have been formulated. Numerical computations for the problem of testing a normal mean with known variance, show that for certain parameter values the MPSPRT procedure does have a lower maximum ASN than Read's PSPRT.

In order to compute the minimum of the ASN at subject to the constraints due to the preassigned error probabilities, an algorithm to find the minimum of a user specified, continuous, single valued function of N-variables subject to K user-specified constraints has been developed. The algorithm, written in FORTRAN-IV and given in the appendix of Chapter-3, is essentially an extension of O'Neill's (1971) algorithm which is based on the method of Nelder and Mead (1965).

Chapter-4 deals with, one more modification of the PSPRT, a two-phase SPRT (TPSPRT) procedure in which Wald's parallel boundaries are broken at some preassigned point of the sample number axis and a region bounded by a pair of diverging lines are used prior to that ; Read's PSPRT can be considered as a special case of such a procedure. The OC and the ASN functions of the procedure have been derived using Anderson's (1960) results. For the problem of testing a normal mean with known variance the proposed TPSPRT procedure is found to be better than Read's PSPRT or the MPSPRT discussed in Chapter-3, as far as the maximum ASN reducing property is concerned.

In the modification of the SPRT (MSPRT) for testing a normal mean with known variance considered in Chapter-5, Wald's parallel boundaries, as in Chapter-4, are broken at some preassigned point of the sample number axis and Anderson's converging boundaries are used prior to that. Thus Wald's, Anderson's and Read's procedures can be studied as special cases of the proposed procedure. As in Chapter-4, the OC and the ASN functions of the MSPRT are derived using Anderson's results. From the numerical examples considered, the procedure has been found to be as good as Anderson's SPRT by virtue of which it is better than the MPSPRT and the TPSPRT discussed in Chapters-3 and 4 respectively.

Chapter-6 contains a modification of Anderson's modified SPRT or the ASPRT, for testing the normal mean with known variance. The proposed modification of the ASPRT (MASPRT) is essentially a partial ASPRT in the sense that after an initial fixed-sample, the MASPRT procedure considers a conditional ASPRT whereas Read's PSPRT considers a conditional SPRT. Unlike the procedures discussed in Chapters 3,4 and 5 the MASPRT is a closed test procedure having an upper bound on the decisive sample number (DSN). For the problem of testing a normal mean with known variance, the OC and the ASN functions of the MASPRT procedure have been derived . For certain parameter values, the MASPRT procedure has been seen to attain lower maximum ASN than that attainable by the ASPRT procedure.

The last chapter of Part-II, Chapter-7, deals with a generalization of the MASPRT procedure discussed in Chapter-6 , In the proposed two-phase procedure (TPASPRT), the continuation region is bounded by a pair of converging lines upto a preassigned point of the sample number axis posterior to which another pair of converging lines is used until the procedure is truncated at a predetermined stage of the experiment. As in Chapters 4, 5 and 6, the OC and the ASN functions of the TPASPRT have been derived using Anderson's (1960) results. The TPASPRT has been seen to attain lower maximum ASN than that attainable by any other known procedure.

It should be mentioned at this point that the five chapters of Part-II have been earlier written in form of papers communicated to, accepted in or published in statistical journals. The original paper forms have been mostly retained except in the discussions of the numerical results. Therefore, certain expressions have appeared in different forms in different chapters. Further it should be pointed out that all the computations in Parts-II and III have been done in the Reyad-1030 computer at IIT-Kharagpur. FORTRAN-IV subprograms have been developed for the computation of OC and ASN of the different procedures discussed in these chapters.

The only chapter of Part-III, Chapter-8, considers the problem of testing the binomial parameter p with various modifications of the SPRT hitherto discussed in this thesis. A general iterative algorithm for computing the OC and the ASN of a modified SPRT having any type of continuation boundaries with or without truncation has been developed. This algorithm is a generalized version of the one proposed by Corneliussen and Ladd (1970) in connexion with testing the binomial parameter. Read's PSPRT, Anderson's modified SPRT, the partial-ASPRT (discused in Chapter-6) and the two-phase-

ASPRT (discussed in Chapter-7) have been studied as special cases of this generalized algorithm. It is seen that the reduction of the maximum ASN becomes more and more pronounced with these procedures in the order mentioned above. Further this chapter also contains a numerical study of a procedure with rectangular hyperbolic boundaries. The numerical study reveals that such a procedure does have the property of reducing the maximum ASN considerably.

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