CHAPTER 1

INTRODUCTION

In the study of the mechanics of deformable bodies the mechanical action of all the external effects upon a body is usually represented by means of body forces and surface tractions. The simplest and most natural forces of this kind are the so-called dead loads which retain their magnitude as well as their initial direction as the body deforms. These forces are conservative in the sense that the work done by them in taking the body from a given configuration to a new one is independent of the path. In general however, the mechanical action of the environment upon a given deformable continuum may not be representable by dead loads. The continuum may interact with the environment and hence may be subjected to forces which depend not only upon the particle positions, but also upon the configuration, velocity field and other kinematical quantities which describe the motion. The action of environment upon the continuum is therefore dependent on the motion and hence the forces which are used to represent this action may be called 'motion-dependent forces'. If the forces that are prescribed in this manner depend only upon the configuration, they are referred as 'configuration-dependent forces'. Similarly the velocity-dependent and acceleration-dependent forces are defined. It is evident that the work done by forces of this kind is, in general, dependent upon the manner in which the body is carried from its initial configuration to a final one, as well as upon these two configurations. Moreover, such forces are not derivable from a potential. For this reason they have been termed not only 'nonconservative forces', but also 'circulatory forces', 'polygenic forces' or simply 'follower forces'. In this thesis the terms 'nonconservative systems', 'circulatory systems' or 'polygenic systems' will be used specifically to refer to 'systems subjected to follower forces'. Thus when the stability of an equilibrium configuration of a finitely-deformed elastic body under follower loads is being sought, one cannot expect to employ the Euler's method in the manner used for dead loads. Here, as has been first demonstrated by Ziegler [190,191], the static approach becomes inadequate. One is, therefore, compelled to adopt a dynamic view of the stability phenomenon with all the added complications which arise from the energy dissipation that accompanies all dynamic processes.

It is interesting to note an observation by Herrmann [49] concerning the reason for studying nonconservative problems. " It is a peculiar feature of stability problems of elastic systems subjected to (nonconservative)follower forces that their analysis arose not out of a desire or need to consider a system which presented itself in engineering practice or in the research laboratory, but rather because the fictitiously applied follower forces acting on a given system were arbitrarily prescribed to depend in a certain manner on the deformation. The motivation for much if not most of the work mentioned in this survey appears to have been shear curiosity in determining the sometimes unexpected behaviour of an imagined system, rather than an explanation of observed phenomena". Although this state of development is concerned with studying the rather unexpected behaviour of 'artificial' nonconservative systems, the area of elastic stability under follower forces is closely related to many practical problems in engineering. The practical significance of stability of nonconservative systems has become increasingly noted with the advancement of modern technology. A flexible missile under thrust, a structural part of an aircraft under aerodynamic forces, flexible pipes conveying fluid, slender cylinders in axial flow (e.g., nuclear reactor fuel assemblies) and elastic systems subjected to impinging fluid jets are obvious examples. Thermally induced flutter of flexible booms is also related to the dynamic behaviour of follower force systems. Herrmann [50] suggests that chemical and electromagnetic energy could constitute appropriate sources which, under suitable conditions of coupling, could induce flutter-type instabilities. Approximate methods are usually required for the solution of such problems. Because boundary value problems in the theory of nonconservative elastic stability are non-self-adjoint, by virtue of the fact that nonconservative forces do not possess potentials, no complete functional exists in the classical form. Hence certain restricted variational principles applicable to nonconservative systems have been developed. A more recent trend has been the interest in the establishment of well-posed variational principles without any constraint conditions by the introduction of the adjoint systems.

This thesis is aimed to present a theoretical investigation of the stability behaviour of nonuniform beams subjected to nonconservative forces with effects of taper, internal and external damping, stiffness of foundation, elastic end support, tangency coefficient, stiffness of shear layer, rotation, tip mass, hub radius and angle of attack. The equations of motion for flexural deformations of nonuniform cantilever beams relative to their undisturbed equilibrium configurations are derived with the aid of a conservation law. The associated non-self-adjoint boundary value problems are determined, and a suitable adjoint variational principle is derived in each case. The stability problems under consideration here are complicated not only by the presence of the internal and external damping terms, but also by the presence of the variable coefficients in the differential equations of motion, which render the process of obtaining the exact solution of the boundary value problems very difficult. Thus, the aforesaid variational principle is used as a basis for determining approximately the complex eigenvalues and hence the critical values of the follower force for prescribed values of the geometric and material properties of the respective systems. The results obtained are compared with those of others wherever possible and a favourable agreement between them has been achieved. The thesis comprises of the following chapters.

Chapter 1 serves as an introductory chapter of the dissertation.

Chapter 2 gives a brief survey of the major papers or books dealing with nonconservative force problems.

In Chapter 3 stability of a tapered cantilever with rectangular cross-section subjected to dissipative and nonconservative forces is studied.

Chapter 4 is devoted to the study of stability of a tapered cantilever with rectangular cross-section on a viscoelastic foundation subjected to a follower force. The effect of internal damping is included.

Chapter 5 is concerned with the stability analysis of a cantilever beam of variable rectangular cross-section resting on a viscoelastic foundation with a shear layer interposed between the beam and the foundation. The beam is subjected to a nonconservative force of constant magnitude at its free end and it is assumed that the materials of the beam and foundation are viscoelastic Kelvin solids.

Influence of an elastic end support on the stability of a nonuniform cantilever subjected to dissipative and nonconservative forces is studied in Chapter 6.

Effects of rotation, tip mass and hub radius on the stability of a nonuniform beam on viscoelastic foundation with a shear layer subjected to a nonconservative force are investigated in Chapter 7.

Chapter 8 deals with the effect of angle of attack on the stability of a rotating nonuniform cantilever with a tip mass subjected to dissipative and nonconservative forces.

Finally, important conclusions based on the work presented in the various chapters of the thesis are summarized in Chapter 9. Scope for further research on the stability of nonconservative force problems is also briefly indicated.

A list of references is given at the end of the thesis. Listings of computer programmes used in connection with the problems treated in the thesis are provided in the Appendix.