1.0 Introduction

This thesis is devoted to studies on some aspects of optimal preventive replacement and spare parts provisioning policies for components (both consumable and repairable) which fail stochastically. Preventive replacement of the equipment (component) is a commonly used method that aims at maximizing the equipment availability or minimizing the total relevant cost per unit time. For finding an optimal preventive replacement strategy, one develops appropriate mathematical optimization model, taking into consideration, the system characteristics and cost parameters. An usual assumption in these models is that a replacement renews the equipment (component). Under this assumption, no distinction need be made between the preventive replacement and preventive repair (overhaul). The decision to repair (overhaul) or replace here logically depends on the degree and nature of degradation of the equipment (component), the costs involved and the available repair facilities. Once a decision at a replacement point is made to repair (overhaul) the equipment, it is to be presumed that it is more economical to repair (overhaul) the equipment than to scrap and to replace it with a new item.

In practice, any equipment has a hierarchical product structure; it is composed of several assemblies/ subassemblies, which in turn are composed of a number of components. Generally, costly assemblies are repaired (overhauled) whereas components are replaced with new ones. Therefore, it is necessary, in practice, to have an optimal policy for the inventory of consumable spares and repairable assemblies/subassemblies inventory. Since the demand for consumable as well as repairable spares is affected by the replacement/repair (overhaul) decision, any decision on replacement and/or repair (overhaul) should be made jointly with the spare provisioning decision. Another related issue which should also be considered jointly with the above two decisions is the number of repair channels to be maintained to repair repairable assemblies/subassemblies, as the level of the repairable inventory depends not only on the rate at which they are removed from the system for repair but also on how quickly they are repaired.

Analytically, finding a jointly optimal preventive replacement/repair (overhaul), spare (repairable and consumable) provisioning and repair capacity decision is very complex and therefore challenging. Some of the important factors which contribute to this analytical complexity are : (i) stochastic nature of time to failure, time to repair and time to procure an equipment and (ii) multi-indenture or father-son type of relationship between the main assembly and its subassemblies and components. Practically, in many large scale industrial activities (for example, integrated steel plants, mines and fertilizer plants, etc.) and military organizations, optimal maintenance, including preventive replacement and spare parts provisioning, is of paramount importance, particularly, in developing countries where capital resources are scarce. In these organizations, a sizeable portion of the inventory investment is in repairable (recoverable) inventory, although percentagewise, most items are consumable. Hence, management of maintenance (including preventive replacement), spare provisioning, and repair capacity decision is practically important and theoretically challenging.

1.1 Maintenance Models

A maintenance model deals with the control and surveillance of a stochastically deteriorating system. For three decades, there has been a large and continuing interest in such models primarily due to their roots in many industrial and military applications and lately due to many new applications in health, ecology and environment. This interest is evident from a large number of papers and books published in the related areas. For comprehensive survey, one may see McCall (1965) and Pierskalla and Voelker (1976).

Published maintenance models can be classified according to the type of problems modelled. Fig. 1.1 shows

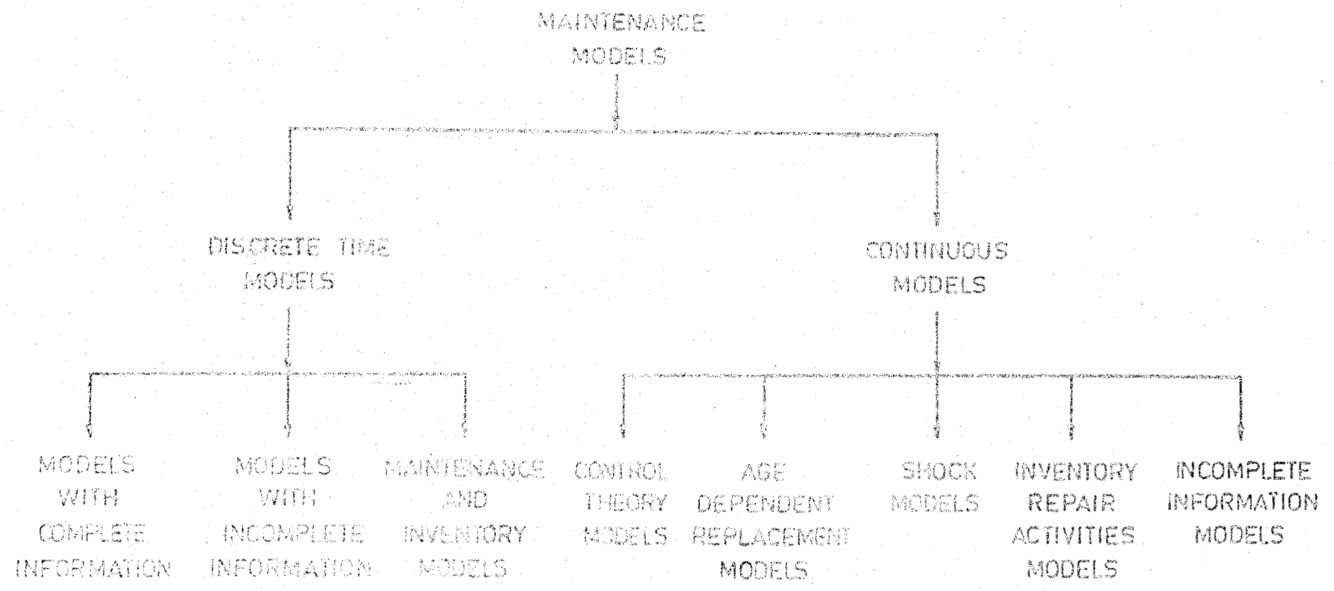

FIG. 1·1 CLASSIFICATION OF MAINTENANCE MODELS one such classification.

Discrete time models select actions at discrete points in time. These models,as they appear in the literature, utilize information regarding the degree of deterioration of the unit(s) in order to select the best repair or maintenance action, including replacement, at certain discrete points in time. In some cases, an inspection must be made to ascertain the state of the system before repair decisions are made, while in others, it is assumed that the state of the system is always known. For incomplete information models, action must be taken under uncertainities about the costs, underlying failure laws, or observations of the state. Inventory models involve periodic restocking of inventory of spare parts. In the formulation of these types of models, the system is treated as a unit and individual components are ignored. Most of these models are based on Markov/semi-Markov decision theory or inventory theory and utilize linear and dynamic programming as primary solution techniques. For details of specific models, one may refer to Gertsbakh (1977), Kolesar (1966), Luss (1976), Rosenfield (1974), Pierskalla and Voelker (1976), and Bosch (1983).

Continuous time models do not restrict maintenance or inspection activities to a set of discrete points in time. Therefore, in these models, the decision maker takes maintenance decisions anywhere on the continuous time axis. His job is essentially to locate finite and discrete number of such decision points on this continuous time axis.

Control theory models assume that the maintenance activity occurs as a continuous stream and the decision maker tries to optimize the rate of maintenance expenditure over a finite or infinite time horizon. Some of the important models which utilize the principles of optimal control theory in maintenance are due to Masse (1962), Naslund (1966), Thompson (1968), Sethi and Morton (1972), Sethi (1973), Tapiero (1973) and Anderson (1981).

Age dependent replacement models are concerned with the replacement of the equipment under consideration on failure and at fixed intervals (block replacement policy) or at age T (age replacement policy). These models attempt to find an optimal replacement interval that minimizes either the total relevant cost per unit time or the total downtime per unit time. The earlier models of age replacement can be found in Barlow and Proschan (1965), Jardine (1973) and Gertsbakh (1977). Since a major portion of this thesis deals with some aspects of age dependent replacement models, a detailed review of the related literature is presented in the next chapter.

In most maintenance models, time to failure is considered to be a random variable and it is assumed to be intrinsic to the unit, under consideration. Some authors, including Taylor (1973), A-Hameed and Proschan (1973) and Feldman (1974, 1975), have taken a different view of the failure process. In their analyses, these authors assume that the unit under consideration is being subjected to exterior shocks, each of which damages or causes wear. The damage or wear process due to shocks is such that the damage or wear accumulates upto a particular time defining the probability of failure at that time. The question of optimal replacement rules in the context of a shock model has received attention recently and is now, an active area of research. Some of the important rules on shock models are due to Nakagawa (1976), Yamada (1980), Zuckerman (1977), Chikte (1981) and Boland and Proschan (1983).

For a system composed of many units, the repair or replacement of one unit should sometimes be considered in conjunction with what happens to the other units. Atleast four different types of policies which exploit the interactions among the units of a system are discussed in the literature. They are opportunistic policies, cannibalization policies, multistage replacement policies and variable repair rate policies.

Opportunistic policies exploit economies of scale in the repair or replacement activity. The fact, that sometimes two or more repair/replacement activities, performed concurrently, cost less than the sum of their individual repair costs, is explicitly considered by these models. So, the necessity of performing atleast one repair might provide the economic justification to do several others at the same time. The details of this type of models are discussed in the next chapter.

In cannibalization and multistage models, units of the same type are utilized at different locations in the system. In response to a failure at a location, an identical unit may be transferred from another location. In multistage models, a new unit always enters into a stage where the failure is more costly and a used unit is introduced in a stage where it is less costly. On the other hand, in cannibalization models, no new units enter into the system. For details of the work on cannibalization models, one may refer to Hirsch et al. (1968), Simon (1970,1972), Rolfe (1970) and Hochberg (1973). Some of the aspects of multistage models may be found in Bartholomew (1963), Naik and Nair (1965a, 1965b), Marathe and Nair (1966) and Jain and Nair (1974).

Lack of complete information regarding (i) the current state of the system, (ii) the time-to-failure distribution of a single unit, and (iii) the cost implication of a particular operating policy, is almost universal in maintenance field. Developing appropriate models for such situations is challenging in its own right. When the state of the system is not exactly known, as in the case of equipments used in emergency conditions, the important decision problem is to find jointly optimal inspection and preventive replacement policy. This type of models have been developed by Savage (1962), Antelman and Savage (1965) and Chitgopekar (1974). The problem of finding minimax replacement policy for situations where the time to failure distribution is unknown, has been discussed by Derman (1961), McCall (1965), Roeloffs (1963, 1967) and Kander and Raviv (1974). Using Bayesian approach, some authors, including Fox (1967), have tried to develop age replacement models for situations where the time to failure is unknown. The problem of developing appropriate models for unknown cost situations has been treated by Dean and Marks (1965) and Elanolt-Johnson (1967).

Many authors (for instance, see Pierskalla and Voelker, 1976) feel that the future research in maintenance modelling should be directed to (i) developing large scale linear and nonlinear models incorporating realistic system constraints and efficiently solving them for important generalizations of Derman's model, age dependent replacement models and maintenance -

salvage models, (ii) developing methodologies to effectively deal with multiechelon and multipart systems, (iii) considering the interaction of maintenance function with related other functions like procurement, and finally, (iv) applying the models of type (i)-(iii) above to real life specific situations. The present thesis aims at bridging some of the gaps cited above. Specifically, this thesis attempts to (i) develop realistic age dependent replacement models that consider the explicit interaction of replacement policy with procurement and stocking of consumable spares, (ii) apply the concepts of age dependent replacement models to complex multicomponent systems with and without consideration of spare provisioning, (iii) the analysis of the behaviour of repairable repair-inventory systems useful in understanding the behaviour of costly and repairable systems, and finally, (iv) finding a jointly optimal replacement and repair-inventory system operating policy for costly repairable systems. The next section gives a birds-eye-view of the specific problems treated in this thesis.

1.2 An Outline of the Thesis

A chapterwise summary of the thesis, presented below, gives an idea of the specific problems treated in this thesis.

Chapter II presents a concise review of the relevant literature so that the contribution of this thesis can be viewed in its correct perspective.

Chapter III is concerned with the problem of finding jointly optimal block replacement and spare provisioning policy for a group of identical equipments. From specific examples, it is shown in this chapter that the jointly optimal block replacement policy is often significantly different from the one where the interaction of spares inventory and replacement policy is neglected. Also included in this chapter is a model for finding jointly optimal group replacement and spare provisioning policy.

Chapter IV deals with a case study on the problem of finding optimal replacement policy for a heavy earthmoving equipment having several stochastically failing components. For preventive replacement of thirty of its critical components, this chapter presents a comparison of four important replacement policies. The policies considered are (i) replacement on failure, (ii) block replacement policy, (iii) group replacement policy and (iv) opportunistic replacement policy. Since, most replacement models found in literature essentially deal with the replacement of single component systems, this case study demonstrates how available replacement models can be fruitfully utilized in finding a viable replacement policy for multi-component systems.

Chapter V of this thesis deals with the problem of finding combined preventive replacement and spare provisioning policy for the same earthmoving equipment considered in chapter IV. Taking into consideration, the peculiarity of the spare procurement system of the organization which uses this earthmoving equipment, jointly optimal replacement and spare provisioning policies have been developed in this chapter. Comparing the minimum total relevant cost (including the spares inventory cost) of the four replacement policies, it shows that opportunistic replacement strategy is the best for the equipment under consideration but the optimal replacement policy for each of them is significantly different from its counterpart where one ignores the spares inventory related costs.

Chapter VI is concerned with the analysis of repair-inventory system. Specifically, it views the repair system as an $M^X/M/n$ queueing system whose input is in batches of repairable units and output is ready for issue units to a pool of usable units. The stock of usable units is also periodically replenished by procuring new units to compensate for any scrapping of repairable units. Assuming the demand for good usable units and the flow of repairable units to be independent Poisson streams, this chapter first develops an optimization model that finds jointly optimal inventory and number of repair channels. A sensitivity analysis is carried out next to study the effect of batch size and other related parameters on the optimal decision.

The optimal repairable inventory policy is derived next under the usual assumption of infinite number of repair channels, therefore constant mean repair time. This model, however. considers the dependence between arrival of repairable units and demand for usable units.

Third model presented in this chapter finds the exact inventory replenishment and number of repair channels considering the effects of (i) the dependence between the arrival of repairable units and the demand for usable units and (ii) the realistic finite number of repair channels. Finally, it compares the above three approaches to repair-inventory systems planning and provides some guidelines on the choice of appropriate model to deal with practical situations.

Chapter VII of this thesis is devoted to the development of a jointly optimal replacement and repair-inventory policy for items which are costly and which are mostly repaired or overhauled on failure or preventive replacement. A close look into this model shows that jointly optimal replacement and repair-inventory decisions are significantly different from the ones where one neglects the interaction between the replacement and repair inventory systems.

Chapter VIII, the final chapter of this thesis, presents a brief summary of the major contributions and limitations of this thesis. This also points out some useful areas of further research relevant to the topics covered in this thesis.