SYNOPSIS

Mathematical models are being extensively used in recent years for various studies of different disciplines. The form and detail of mathematical models and the processes by which they are derived are, in most cases, determined on the basis of the objectives of the study. One can differentiate between the types of models according to their use : descriptive, predictive and explanatory. The descriptive use of mathematical models is the expression of quantitative relationships in terms of equations; the predictive use of the model is to predict the response of an organism to a loss of function in an organ, while the explanatory power of a mathematical model lies in the description it gives of the ways in which different features of system behaviour and structure depend upon each other.

Models are formulated on the basis of the knowledge about the system. This basis may be, i) empirical, ii) theoretical and iii) a combination of empirical and theoretical knowledge. Empirical models are those for which no a priori knowledge about the system is assumed, while theoretical models are necessarily based upon validated a priori knowledge about the system.

The principles of Continuum Mechanics are now a days, being successfully applied by a good number of investigators to explain various physiological processes ranging from mechanisms occurring in the microscopic cells, to complex interactions at the macroscopic level involving whole organs. Many attempts have been made with a view to formulating mathematical models of large physiological systems, e.g. the cardiovascular system including the heart and its circulatory process, the respiratory system, the renal system and the head-neck system. Within the spectrum of models formulated, depending on the purpose of consideration of the model and the extent to which a priori physiological knowledge is available, a variety of levels of conceptual and mathematical representation is likely to exist.

The present thesis aims at illustrating the application of the principles of Continuum Mechanics to a few physiological problems, through the use of mathematical models. The thesis contains six chapters of which the first one is introductory. An attempt is made to include in this chapter, brief discussions on various related physiological and mechanical concepts including importance of mathematical models in physiology, relation between Bio-mechanics and physiology, circulatory system, properties and flow characteristics of blood, mechanical properties of soft tissues in general and vessel walls in particular, equations governing the motion of blood, heart - its structure and functions, mechanical behaviour of cardiac muscles, ventricular aneurysms, head - its structure and functions, mechanical characteristics of various constituents of head, effect of shock and vibration, different categories of head injury and various postulates on brain injury. This chapter concludes with a brief account of earlier investigations related to the thesis.

The second chapter deals with the analysis of a mathematical model for studying the unsteady flows in blood vassels. It is well-known that physiological pumps produce flows by alternate contraction and expansion of the vessel. When muscles start squeezing the vessel wall, the valve at the upstream end gets closed and that at the downstream end becomes open; as a result of this, blood is pumped out in the downstream direction. The main part of the cardiovascular pump is a valved vessel. During systole, when the blood in the left ventricle is forced into the aorta, the mitral valve becomes closed while the atrioventricular valve becomes open. In a situation such as this, the left ventricle forms a vessel with one end closed. For the study of such a stage, a model consisting of a fluid-filled long cylindrical pipe with one end closed by a compliant membrane (which prevents the axial motion of the fluid, leaving radial motion completely unrestricted) has been considered. An exact solution of the Navier-Stokes equation for the unsteady flow of a second-order fluid is presented in this chapter. The derived analytical expressions are computed numerically. The computed values indicate that in the case of low Reynolds number, the effect of fluid viscoelasticity appears in higher order terms. It is further observed that at very low Reynolds number, the distribution of the axial velocity is parabolic and is independent of the elastic parameter. Moreover, the shear stress and frictional drag on the wall are found to depend on the elastic parameter in higher order terms.

By considering the eccentricity of the ventricular geometry, a mathematical analysis for the mechanical behaviour of aneurysms in the left ventricle has been presented in the third chapter of the thesis. The analysis is carried out by considering the constitutive relations, the incompressibility condition and the force-equilibrium equations . Numerical values of some quantities characterizing the behaviour of the damaged region, are also obtained by employing numerical techniques.

In the fourth chapter, an analytical study on cardiac contraction, by modelling the heart as a cardioide of revolution, has been incorporated. The study is based on the assumption that during systole, the interstitial pressure is negligibly small compared to the left ventricular pressure. It is further assumed that the shear stress on the outer surface of the heart is zero except in the region where the right ventricle wraps around the left ventricle. The results indicate that the increased diastolic filling will contract the ventricle more forcefully and eject a larger amount of blood. This is in complete agreement with the Frank-Starling principle. A lemmiscate of revolution is also investigated as another possible model.

Two models for a human-sized head, are of concern in the two subseqent chapters. In the analysis presented in the fifth chapter, the skull is considered as an anisotropic spherical shell and the brain matter is represented as an inviscid compressible fluid. The effect of a translational acceleration considered as a general function of time, is studied in particular. The method of Laplace transform is used to achieve the analytical solution of the problem. The stress distribution in the system is studied by numerically computing the derived analytical expressions.

In the sixth and the concluding chapter, the problem of wave propagation in a head, generated by a local axisymmetric impact on its outer surface, is considered. As in the earlier chapter, the skull is considered as an elastic spherical shell but in this analysis the viscoelasticity of the brain matter (as per previous experimental observations) has been paid due attention. The skull material is treated as isotropic and

homogeneous but the presence of the cerebrospinal fluid (considered as a compressible but inviscid irrotational fluid) inside the cranium has been accounted for in the analysis. Some numerical results obtained on the basis of the analytical study, are also presented.