CHAPTER 0

SYNOPSIS

0.1 INTRODUCTION

Analogous to traditional single objective optimization, many real life problems involve the optimization of several conflicting objectives. Such systems with multiple conflicting and noncommensurable criteria are termed as Multi-criteria decision systems.

Management Scientists and System Analysists are fully aware of the multicriterion nature to be faced in the real world decision problems. Consequently, there has been a growing interest in the development of the methods of formulation and solution of Multiple criteria decision making problems over the last two decades.

In complex systems a good decision (solution) is feasible only when multiplicity of objectives are taken into account simultaneously. The concept of unique optimization no longer holds to optimize simultaneously all the objectives which are normally conflicting to each other. This is therefore replaced by using the concept of best satisfying solution, in some way, to the decision maker[DM]. Obviously, the best satisfying solution should be understood in the sense that no alternative better than this exists.

In an attempt to search for the best satisfying solution, Charnes and Cooper (1961) have introduced the concept of goal programming (GP). A goal is defined as an objective augmented with an aspiration level stated in terms of its attribute. It is the desire of the DM based on his past experience or otherwise which the analyst has to keep in his mind while developing the solution. Goal programming is a method applied to solve multiobjective optimization problems with the aim of finding solutions which satisfy, as close as possible, the DM's desired aspirations. Another class of methods called Interactive Methods(which also give satisfying solution), involve progressive definition of the DM's preferences through a series of DM - Analyst or DM - Machine interactions. At each such interaction the DM provides his preference or trade-off information based on the current solution. A new solution is next generated based on these informations.

In most of the realistic decision situations the various parameters involved to define a goal are not precise. For example, statements such as 'the profit on product A is very less' are often encountered. To deal with such situations, Fuzzy Set theory as introduced by Zadeh (1965) comes in handy which provides gradual transition from the realm of rigorous, quantitative and precise phenomena to those that can only be articulated by vague, qualitative and imprecise concepts. The use of Fuzzy set theoretic concepts in goal programming is termed as FUZZY GOAL PROGRAMMING (FGP). This approach has been initiated by Narasimhan(1980) and over the years it has become an important area of study in Operations Research.

A Goal with inexact target value is called a fuzzy goal. A fuzzy goal programming problem with m imprecise goals may be stated as follows:

$$
\begin{array}{lrcl}
\text{Find } X & & & \\
\text{to satisfy} & G_1(X) & \gtrsim & g_1 \\
 & G_2(X) & \gtrsim & g_2 \\
 & \cdots & & \cdots \\
 & \cdots & & \cdots \\
 & G_h(X) & \cong & g_h \\
 & \cdots & & \cdots \\
 & \cdots & & \cdots \\
 & G_i(X) & \lesssim & g_i \\
 & \cdots & & \cdots \\
 & G_m(X) & \lesssim & g_m \\
 & X & \geq & 0
\end{array}
$$

where the symbol " $\sim$ " refers to the fuzzification of the aspiration level. For example, $G_1(X) \gtrsim g_1$ means the objective $G_1(X)$ to be achieved at a level which is approximately greater than or equal to the value g_1. This fuzzy model is then converted into a crisp one by designing suitable membership functions for the objectives and using relevant aggregation operators which aggregate the membership functions so that the conventional algorithms can be used to solve the problem.

0.2 OBJECTIVES

The objectives of the present study are:

1. To give a brief critical appraisal (state-of-the art) of the methodologies for tackling fuzzy goal programming problems (Chapter I).
2. TO discuss the fuzzy and nonfuzzy priorities which influence the fuzzy goals in formulation as well as the solution procedures. Also the FGP problem with nonfuzzy priorities using intersection operator is discussed (Chapter II).
3. To investigate the additive model of FGP for equivalent and nonequivalent fuzzy goals (Chapter III).
4. To devise an interactive procedure in FGP format without weights and priorities for exploring the efficient frontier by using membership space to arrive at a best satisfying solution of a FGP problem (Chapter IV).
5. To introduce the concept of 'efficiency' called 'fuzzy eficiency' and a modified linear membership function for a FGP (Chapter V).

6. To discuss the applications of FGP models in the Aggregate production plannning, Developing low cost high protein food and Sales effort allocation problems (Chapter VI).

0.3 ORGANIZATION OF THE THESIS

Chapter I presents a brief state-of-the-art on FGP and its closely related areas. The basic concepts and notations are first explained. Literature survey on multiple objective decision theory and goal programming is next made. Some elements of fuzzy set theory as much as required for the investigations made in this thesis are then given. Finally, an exhaustive critical review on FGP is presented.

Chapter II deals, with priority structure in FGP. The existence of more goals than one usually necessitates the introduction of priority structure. Narasimhan(1980) has discussed the fuzzy priorities such as 'very important', 'moderately important', 'important' etc., and used composite membership function of fuzzy goals to tackle the impreciseness involved in the expressions of priorities. We have investigated this problem using the fuzzy priorities and formulated a FGP problem with preemptive priority structure. We have found that the solution is computationally efficient. In preemptive priority structure the goals are ranked according to the rule if r<s then goal (a subset of goals) in the rth priority

level holds higher priority than the goal (a subset of goals) in the sth priority level. The main idea behind our ivestigation is that, even though the fuzzy priorities are expressed verbally as 'very important', 'moderately important', 'important' etc., and therefore fuzzy in nature they can be considered correspondingly in a preemptive order 'first', 'second', 'third' etc. (This work is published in Fuzzy Sets and Systems under the title "Priority Structure in Fuzzy Goal Programming", vol. 19, p. 251 - 259, 1986).

In Chapter III, an Additive model for FGP is discussed. This model uses the arithmetic addition as an operator to aggregate the fuzzy goals to construct the relevant decision function. Cardinal weights and ordinal ranking for nonequivalent fuzzy goals are also incorporated in the model to reflect more of the reality of the situation. The solution procedure is illustrated with a numerical example. (The contents of this chapter have been accepted for publication in Fuzzy Sets and Systems).

In Chapter IV, an interactive procedure is proposed to solve FGP problems. Satisfaction of the DM in terms of membership functions of the fuzzy goals is considered. This procedure is developed keeping in view the following points: 1) the DM need not specify any weighting structure, 2) the analyst has to search for a satisfactory solution and 3) at each interaction the analyst extracts information from the DM whether the solution is satisfactory. If not satisfactory, the analyst elicits howmuch the DM concedes from the most satisfactory fuzzy goals of the current solution in terms of their membership functions in order to improve the degree of satisfaction of the other fuzzy goals. A numerical illustrative example is finally presented in this chapter.

In chapter V the concept of fuzzy efficiency is first introduced. Relevant definitions are coined and a theorem for exploring fuzzy efficient solution is established. The method may be applied to test the efficiency of the solutions obtained in other chapters also. A modified linear membership function for FGP is also introduced and discussed in this chapter.

In Chapter VI, FGP models of aggregate production planning, low cost high protein food and sales effort allocation are presented.

FGP model of aggregate production planning problem is for multi product - single period case. The fuzzy goals considered are: i) the satisfaction of the fluctuating market demand, ii) the manager's imprecise limitation in in-process inventory, iii) Overtime operations in machine centres, and iv) to attain the desired profit. In low cost high protein food problem, the nutritional requirements which cannot be precribed precisely are considered as fuzzy goals while developing low cost high protein food through a suitable blending of available cereals and pulses. The FGP model of sales effort allocation includes the fuzzy goals of sales revenue, overtime limitation and advertising expenditure.

In chapter VII, concluding remarks are made and the scope for future investigations is discussed.