INTRODUCTION AND SUMMARY

The dynamics of many control systems is described by high order differential equations. However, the behaviour is governed by a few dominant parameters, a relatively minor role being played by the remaining parameters such as small time constants, masses, moments of inertia, inductances and capacitances. The presence of these "parasitic" parameters is often the source for the increased order and the "stiffness" of the system. The "curse" of this dimensionality coupled with stiffness causes formidable computational difficulties for the analysis and control of such large systems. The singular perturbation method using the reduced order models and relieving the stiffness, is a "gift" to control engineers. As such it is very attractive to formulate many control problems to fit into the framework of the mathematical theory of singular perturbations which has a rich literature [1-3]. The singular perturbation theory in continuous control systems has reached a certain level of maturity and is well documented [4-8].

Discrete systems are very much prevalent in science and engineering. There are three important sources of discrete models described by high order difference equations containing several small parameters [9]. The first source is digital simulation, where the ordinary differential equations are approximated by the corresponding difference equations [10]. The study of sampled-data control systems and computer-based adaptive control systems leads in a natural way to another source of discrete-time models [11,12]. Finally, many economic, biological and sociological systems are represented by discrete models [13,14]. In spite of its paramount importance, the area of singular perturbations in difference equations and its applications to discrete control problems has not so far received sufficient attention [15-25]. It is in this context that the present investigation is taken up in the field of singular perturbation analysis of discrete control systems. The motivation for the present investigation comes mainly from the work of Comstock and Hsiao [15]. The thesis starts with the analysis of singularly perturbed difference equations in classical form and the various state space models [26-29] and contributes towards the development of singular perturbation methods for open-loop control [30], closed-loop optimal control [31], and the Kalman filter in discrete systems [32]. Various implications in casting the equations in a form suitable for singular perturbation analysis and many distinguishing features of the analysis are examined with relevance to each problem. Typical numerical exampled are provided to illustrate the proposed methods.

The thesis is organised as follows:

In Chapter 1, the singularly perturbed difference equations in classical form are formulated as initial value problems and boundary value problems. Methods are developed to obtain approximate solutions in terms of an "outer series" based on the degenerate (reduced) problem and a "boundary layer correction series" obtained by using certain transformations on the original problem.

In Chapter 2, we consider the state space modelling and analysis of singularly perturbed difference equations in order to give a general framework suitable for control engineers. Depending on the position of the small parameter, three state space discrete models are formulated and techniques are developed to obtain approximate series solutions [26-29]. The computational requirements of singularly perturbed differential equations are tremendous due to their stiffness [33-38]. A method is therefore suggested to cast singularly perturbed differential equations into the corresponding discrete models. The case of sampled-data control systems is also examined.

In Chapter 3, the three-time scale property of difference equations is first examined. Then, the open-loop optimal control of singularly perturbed discrete system is investigated [39]. For the resulting two-point boundary value problem, a method is developed consisting of an outer series and two correction series corresponding to "initial" and "final" boundary layers [30].

In Chapter 4, we investigate the basic ideas of the singularly perturbed nonlinear difference equations. Then the closed-loop optimal control of a linear singularly perturbed discrete system is examined [39,40]. For solving the resulting matrix Riccati difference equation, a method is developed. The steady state solution of the Riccati equation is also examined [31].

Finally, in Chapter 5, we consider the singularly perturbed discrete system with stochastic environment [41]. The controlling equations for the Kalman filter are formulated.

A method is proposed which gives a considerable reduction in computational complexity [42]. The case of steady state solution is also examined [32].