Modal Damping Prediction for Vibrating Solids: Constitutive Models and Finite Element Computations

Abstract

First principles prediction of the internal vibration damping of engineering components is not routine in finite element packages. With such predictions, designers would be able to assess the noise and vibration levels in engineering systems right from the component design stage. To this end, we revisit the modelling of internal material dissipation under spatially variable triaxial stresses, a research topic that peaked five decades ago but bears reexamination in view of modern computational power.

Internal energy dissipation in many materials, per stress cycle and per unit volume, is known to be frequency-independent and proportional to some power ($m \geq 2$) of a suitable equivalent stress amplitude ($D = \xi \sigma_{eq}^m$). Definition of this equivalent stress amplitude under arbitrary triaxial stress states remains an open question. Such a definition is needed for computing modal damping of arbitrary solid bodies using finite element packages.

In this thesis we develop macroscopic damping constitutive relations, for arbitrary triaxial time-harmonic stresses, by first considering numerous randomly dispersed microscopic dissipation sites. The resulting dissipation models are used in finite element prediction of modal damping in solid objects.

Constrained by empirical evidence, we consider two mathematically simple rate-independent dissipative phenomena in our assumed micromechanical dissipation models: (i) Coulomb friction, and (ii) ambient-temperature plasticity. The first case is a flat crack in an elastic material, with Coulomb friction between the crack faces. The second case involves dissipation due to microscopic elasto-plastic flaws.

For the first model, the macroscopic dissipation is obtained by Monte Carlo averaging of the dissipation from many randomly oriented non-interacting microcracks, and finally fitted using a multivariate polynomial. In the second model, namely distributed microscopic elasto-plastic flaws, we find two limiting special cases that are analytically tractable: spherical flaws, and flaws that are flat and thin. In each of these cases we assume a random distribution of flaw strengths and orientations, and a formula for the macroscopic dissipation is obtained analytically (for spherical flaws) or semi-analytically (for flat and thin flaws). For spherical flaws, the averaged dissipation is governed exactly by the distortional strain energy. For flat and thin flaws, when m is between 2 and about 6, the net dissipation is accurately, but not in general exactly, described by a power of the distortional strain energy.

We finally suggest that for engineering modeling purposes of metallic structures, for moderate m, a simple power law based on the distortional strain energy might be reasonable. We demonstrate use of this dissipation model for finite element computation of the modal damping ratios of arbitrary solid objects using both solid and shell elements in ANSYS. The results are then verified with known analytical results for the cases of several analytically tractable geometries. An interesting aspect of these damping results is that the modal damping ratios show a variation of over one order of magnitude over the cases we have considered. Torsion dominated modes have high damping and the purely radial mode of a solid sphere has low damping. We also verify that, at least in some cases, deliberately induced stress concentrations can lead to improved damping for materials where $m > 2$.

Keywords: Vibration damping, internal dissipation, frictional microcrack, Monte Carlo, elasto-plastic flaw, inclusion, plasticity, distortional strain energy, effective damping ratio