CHAPTER 0

INTRODUCTION

0.1. Some Standard Techniques of Quality Control

Quality control techniques have two major functions in a manufacturing process. One is to maintain the process in a state of control continuously with time, and the other is to ensure that the output of the process conforms to certain optimum specifications. The main tools of quality control are various kinds of control charts and acceptance sampling procedures.

A manufacturing process is considered to be in a state of 'statistical control' when assignable variation, that is variation ascribable to any particular source, is absent. A control chart is basically a graphical device for detecting from time to time the presence of such variation in the quality of items produced.

The standard practice (Shewhart, 1931) is to determine the value x_i of some quality characteristic (such as the mean, standard deviations, fraction defective etc.) of a sample taken at time t_i, $i=1,2,\ldots$

and to plot the x_i's on a chart marked with upper and lower action limits. So long as the plotted points remain between the two action limits, the process is regarded as in a state of statistical control. Lack of control is detected as soon as a point falls beyond either action limit and corrective action is called for in that situation. The upper and lower action limits usually correspond to $E(x_i) + 3SE(x_i)$ and $E(x_i) - 3SE(x_i)$ respectively, where $E(x_i)$ is the expected value and $SE(x_i)$ is the standard error of x_i. The justification for these so called 'three sigma' limits is mainly empirical: it has been found from experience that they control both the first kind of error (detection of assignable variation when it is not present) and the second kind of error (failure to detect assignable variation when it is present) quite effectively in most situations.

The standard control chart procedure described above however, suffers from one important defect which may substantially affect its power of detecting changes produced by sources of assignable variation. At any point of time t_i, it takes into account the value of only one observation x_i, completely neglecting the information provided by observations $x_{i-1}, x_{i-2}, \ldots$ taken earlier. This information, however, may sometimes be quite useful in detecting real changes in the process. Bartlett (1953), Duncan (1956) and Cowden (1957) have shown how a run of successive points above or below the target value can indicate the presence of assignable variation even when all the points are within the action limits. Thus, even a rather indirect use of several successive observations instead of only one may increase the sensitivity of the Shewhart chart. A more direct use of all the available observations however, does not seem to be possible on a Shewhart chart.

Page (1954) and Barnard (1959) have suggested a new type of control chart called the cusum chart, in which the cumulative sum of all observations upto a point of time is plotted instead of a single observation. The cusum chart, which thus makes full use of all available observations, appears to detect assignable variation much more quickly than a standard control chart. A good deal of recent work on quality control is concerned with construction of different types of cusum charts for use in different situations. A review of some important work on this topic is given in section 0.2.1.

0.2. Review of Some Previous Work

0.2.1. Cumulative sum control charts

Page (1954) introduced the cumulative sum control chart as an alternative to the standard (Shewhart) chart for controlling a process parameter θ at some desired or target value θ_0. In these charts, we plot the sum

$$S_m = \sum_{i=1}^{m} (x_i - k)$$

against the sample number m $(m = 1, 2, \ldots)$ where the observation x_i denotes some suitable estimator of θ calculated from the ith sample and k a properly chosen 'reference value' related to the target value θ_0.

Page (1961) has described a procedure for controlling the variation of the parameter in one direction only, say upwards. According to this procedure, a lack of control is indicated when and only when a plotted point S_m exceeds the lowest plotted point by an amount greater than h, a predetermined positive quantity called the decision interval. He has indicated how a suitable choice of k and h can be made.

The procedure for controlling two-sided variation of the process parameter by a cusum chart with the help of a V- mask has been outlined by Barnard (1959) and Page (1961).

The V- mask (see figure 0.1) has two parameters (i) the lead distance $d = OP$, representing the distance between the last plotted point O and the vertex of the mask P, and (ii) the angle of the mask $\phi = \angle OPQ$, giving the slope of a limb of the V- mask to the horizontal axis. A lack of control is indicated if any of the points plotted before O falls below the line PQ. Barnard (1959) and Page (1961) have not given any mathematical method for determining the values of mask parameters. Johnson and Leone (1962), however, have established a relationship between the cusum chart technique and Wald's sequential probability ratio test by assuming that the probability of second kind of error is small. From this relationship, they have derived mathematical formulae for determining the mask parameters d and ϕ.

Johnson and Leone (1962) have constructed cusum charts for the mean and variance of a normal distribution as substitute for Shewhart's $\bar{X}$ and σ charts, as well as cusum charts for the fraction defective and number of defects, which would be used instead of Shewhart's p and c charts.

Johnson (1963) gave the construction of the cusum charts for the mean when the underlying distribution of the variable is folded normal. In 1966, he described the construction of the cusum chart for the mean of the Weibull distribution.

0.3. Problems under Investigation

Part One (Chapters 1 - 4) of the present thesis is devoted to the study of certain cumulative sum control charts, while Part Two (Chapters 5 - 8) deals with some acceptance sampling procedures. Part Three (Chapters 9 - 11) contain two problems on estimation of the parameters of certain distributions and a problem on optimization of sample size and sampling interval in an $\bar{x}$-chart.

In Chapter 1, cumulative sum control charts for the mean and the variance of a doubly truncated normal distribution have been constructed by generalising the procedure given by Johnson and Leone (1962). The formulae for the mask parameters have been obtained. An expression has been derived for the average run length which is defined (Barnard, 1959) as the average number of samples taken before a given change is detected in the process parameter. The mask parameters and the average run lengths have been tabulated for certain truncation points and for specific changes in the process parameter. It has been found that the average run length and the lead distance decrease, while the angle of the mask increases with increasing range of truncation.

In Chapter 2, a cumulative sum chart for controlling the mean of a gamma distribution has been constructed. The average run length of the cusum chart for gamma distribution has been compared with that of the corresponding Shewhart chart. The chart for exponential distribution has been obtained as a special case of that for the gamma distribution. An investigation has been made about the error involved when a gamma distribution is approximated by an exponential distribution in constructing a sequential probability ratio test for the mean.

In Chapter 3, cumulative sum charts have been constructed for observations grouped in a frequency distribution assuming the underlying population to be normal. The formulae for mask parameters and the average run lengths have been derived. The mask parameters and the average run lengths have been tabulated for certain values of h , the width of class interval. It has been found that the average run length, the lead distance and the angle of the mask increase considerably as h increases.

In Chapter 4, cumulative sum control charts for the normal and folded normal distributions have been constructed assuming the observed variance to be different from the population variance. The formulae for the mask parameters and the average run lengths have been obtained. The mask parameters and the average run lengths have been tabulated for certain ratios of observed to population variances. It has been found that mask parameters and average run lengths increase as the ratio increases.

In Chapter 5, the robustness of a continuous sampling inspection plan based on cumulative sum chart for the mean has been investigated. An expression for the average run length for non-normal distribution has been obtained and has been tabulated for certain values of skewness and kurtosis coefficients. It has been found that the average run length of 'normal theory' sampling plans is affected largely if the populations are leptokurtic and positively skewed, while negative skewness seems to have a less deterrent effect.

In Chapter 6, a continuous acceptance sampling procedure based on cumulative sum chart for the mean has been described. The Operating Characteristic (OC) function of the plans has been obtained. An investigation has been made about the effect of non-normality on the operating characteristic function of the plans. It has been found that for high values of skewness and kurtosis coefficients the OC function is considerably affected.

In Chapter 7, following the lines of Greb (1963), sequential sampling schemes have been developed for which the plan-constants are the functions of the Average Outgoing Quality Limit and the Acceptable Quality Level. Some graphs have been provided from which sampling plans can be formulated for specific values of the Average Outgoing Quality Limit and the Acceptable Quality Level.

In Chapter 8, sequential sampling procedures based on the Acceptable Quality Level (AQL) and the Outgoing Quality Probability Limit (OQPL) have been proposed. Some graphs have been provided by which a sampling plan can be constructed for given values of the Outgoing Quality Probability Limit and the Acceptable Quality Level.

In Chapter 9, the maximum likelihood estimate of the parameter of an exponential distribution has been obtained when the observations are grouped in a frequency distribution. An expression has been derived for the efficiency of the estimator compared to the corresponding one based on ungrouped observations.

In Chapter 10, the statistical properties of certain compound distributions (namely, the Poisson-Gamma, the Poisson-Beta and the Pascal-Rectangular) have been studied. The parameters of Poisson-Gamma, Poisson-Beta and Pascal-Rectangular have been estimated by the method of moments.

In Chapter 11, the optimum sample size n and sampling interval h have been determined for an $\bar{X}$-chart when the underlying population is represented by an Edgeworth series. The values of n and h have been tabulated for certain values of skewness and kurtosis coefficients. It has been found that 'normal theory' values do not undergo much change for moderately non-normal populations.