SYNOPSIS

Excessive vibrations in any machine or structure are generally regarded to be undesirable. The vibrations of structural members can not be avoided when the excitation is due to shock or is random over a wide frequency range. The tendency of structures to respond vigorously to all manner of excitations is aggravated by the trends towards light weight and unit construction as in aerospace industries. The problems of severe vibration are chronic in missiles, aircraft, and ships where excessive vibrations can lead to failures by fatigue.

The severe vibrations of structural members can be reduced, however, by dissipation of energy within the vibrating members themselves. The long chain molecule polymers can be used with advantage because their imperfect elasticity gives much larger energy dissipation during deformation as compared with metals. Such materials are sometimes refered to as being viscoelastic. A combination

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A part of the work reported in this thesis appears in published literatures as given below.

1. Sharma, S.R. and Rao, D.K., 'Static Deflections and stresses in Sandwich Beams under Various Boundary Conditions', Journal Mechanical Engineering Science, Vol.24,No.1,Mar.1982,pp.11-20.

2. Sharma,S.R. and Rao,D.K., 'Static Analysis of Three-Layered Sandwich Cantilever Beams', Paper No.DE-3, to appear in Journal of Vibrations, Acoustics, Stress and Reliability in Design, Trans.ASME, 1983.

of metal and polymer damping material layers will ideally provide strength and rigidity but with a low response to vibrations. Such an arrangement consisting of alternate elastic and viscoelastic layers is called a sandwich treatment.

In engineering applications, sandwich beams are used where high strength and light weight are desired and where damping is necessary to dissipate vibratory energy. In particular, a designer will be interested in knowing how the strength of an existing structural layer is affected by applying sandwich treatment to it. This implies that he must know how the maximum deflection of a treated beam compares with that of an untreated beam for the same load. With this knowledge, he can proceed to determine an optimal sandwich structure which has essentially the same resistance to deflection as the original one that it is going to replace, and has, in addition, a good vibratory energy dissipating capacity built into it.

The objectives of designing a sandwich structure, as given above, are well known. However, the needed design information, such as expressions for static deflections and stresses, etc. are not available for most of the boundary conditions of engineering interest. It is evident from the review of literature undertaken in connection with the present study that static and dynamic analysis of various types of sandwich cantilever beams such as those having a Clamped, but unrestrained edge, has not been done so far.

Also very little attention has been paid to the study of the deflection and thickness-shear modal patterns of vibration of sandwich cantilever beams.

The aim of the study presented in this thesis is broadly as follows :

1. To derive comprehensive, exact expressions for the static deflections and stresses in sandwich beam for various practically important boundary conditions for both uniform as well as concentrated loads.

2. To estimate the loss factors and the corresponding resonant frequencies when the above mentioned structures are subjected to vibrations.

3. To study the deflection and thickness-shear modal patterns of vibration of three-layered and multi-layered sandwich cantilever beams.

In the first phase of the study, a detailed analysis of static deflections and stresses in three-layered sandwich cantilever beams is carried out. Three types of boundary conditions dealing with the mechanism of 'Clamping' at one edge and generation of 'Free' condition at the other edge are investigated. The three types of cantilever beams investigated here are as follows : (1) One edge Clamped and the other edge Free (C-F), (2) One edge Clamped and the other Free edge riveted ($C-F_r$), (3) One edge Clamped, but unrestrained and the other edge Free (C_u-F).

The difference between a riveted and an unriveted edge is explained below :

(a) Riveted edge. In this case a rigid rivet is applied through the edge-section of the beam. This prevents shearing distortion of the sandwich core, and hence the edge section of the beam always remains plane.

(b) Unriveted edge. In this case the longitudinal motion of face layers is not restricted so that the edge section no longer remains plane.

In the case of a Clamped, but unrestrained edge(C_u) only the structural layer is clamped. To this structural layer, the damping layer is bonded, which in turn is glued to the constraining layer. A small gap is provided between the support of the Clamped edge and the end faces of the damping and constraining layers so that their distortions are not restricted during loading.

The differential equations governing the static deflection of the sandwich beam are derived using the variational method in the first phase. These equations are then solved for deflections and stresses by using the Laplace Transform method. The effects of geometric and shear parameters on deflections and stresses have been illustrated with the help of non-dimensional graphs. These graphs show how the mechanism of 'Clamping' at one edge and generation of 'Free' edge condition at the other edge can be utilised to increase the stiffness of sandwich cantilever beams.

In the second phase of the study, exact expressions for deflections and stresses in three-layered sandwich beams having following boundary conditions are derived for both uniform as well as concentrated loads :

(1) Both edges Pinned (P-P), (2) One edge Pinned and the other edge Pinned and riveted ($P-P_r$), (3) Both edges Pinned and riveted (P_r-P_r), (4) One edge Clamped and the other edge Pinned (C-P), (5) One edge Clamped and the other edge Pinned and riveted ($C-P_r$), (6) Both edges Clamped (C-C).

The influence of system parameters on deflection and stresses is illustrated by means of exact formulae and graphs. These investigations reveal that riveting an edge of sandwich beams can appreciably reduce the deflections and stresses.

In the third phase of the study, Hamilton's principle is used for obtaining the differential equations of motion alongwith boundary conditions governing the free vibration of sandwich beams. The equations are then solved exactly by using the Laplace Transform approach. Expressions for complex frequency factors of three-layered sandwich beams are derived for nine boundary conditions mentioned earlier. The loss factors are found from the complex frequency factors. Sandwich cantilever beams having three different types of boundary conditions (i.e., C-F, $C-F_r$ and C_u-F) are selected for the analysis.

The effects of geometric and shear parameters as well as boundary conditions on frequency and loss parameters, are illustrated with the help of graphs. The analysis of results indicates that an increase in geometric parameter results in an increase in both the loss and frequency parameters. It is also seen that riveting of Free edge of sandwich cantilever beam has a beneficial effect in increasing the damping for Y=5 and $g>8$.

In the fourth phase of the study, the deflection and thickness-shear modal patterns of vibration of different types of three-layered and five-layered sandwich cantilever beams (i.e., C-F, $C-F_r$ and C_u-F) are analysed in detail. The equations of motion of multi-layered sandwich beams are derived taking into consideration higher order effects like rotatory inertia, bending, and extension of the cores. No restrictive assumptions are made regarding the bending and shear rigidities of the cores. The resonant frequencies and loss factors are computed by an approximate variational method of replacing the state of the beam by a series of known approximating functions.

A study of the deflection and thickness-shear modal patterns reveals that in I mode of vibration the deflection pattern is dominant whereas in II and III modes the thickness-shear patterns are dominant. It is also seen that for $C-F_r$ beams there is no axial movement of the layer cross-sections at the riveted edge.

In the fifth phase of the study, loss factors and the corresponding frequencies are found for three-layered and five-layered sandwich cantilever beams having three different types of boundary conditions. The boundary conditions considered here are : (1) C-F, (2) C_u-F, (3) C-F_r. These boundary conditions have been explained earlier.

The analysis of results reveals that the maximum damping is attained in the case of C_u-F beams with constraining layer thickness of 2 mm. It is also seen that damping is adversely affected by an increase in the constraining layer thickness.

In the last phase of the study, the various directions in which the work can be extended are indicated. An appendix at the end gives the computer programs developed for solving the different types of problems discussed earlier.