CHAPTER I

INTRODUCTION

Flows of real fluid occurring in nature are generally turbulent in view of the large scale of the motion. It is found that the laminar motion occurs at such a low Reynolds number that any deviations from it are damped out by viscosity. At sufficiently large Reynolds numbers the fluid motion becomes unstable. This implies the inability of the basic flow to sustain against perturbations to which the flow is subjected. If, on the other hand, the perturbations decay with time and the flow reverts to the original state, the flow is said to be stable.

There are essentially two distinct methods for studying the stability of flows. In the first method, the disturbances superimposed on the basic flow are assumed to be infinitesimal. This enables one to linearize the relevant equations and study them by the technique of normal mode. A comprehensive account of this technique has been given in the monograph by Chandrasekhar [1]. The second approach is based on the energy method due to Serrin [2]. In this method, the time rate of change of the total kinetic energy of the disturbance superimposed on the basic flow is studied, the amplitude of the disturbance being assumed finite. If the total kinetic energy decreases with time, we say that the flow is stable. Utilizing this method one may obtain a universal stability estimate which provides sufficient condition for asymptotic stability in the mean. The linear stability theory gives an upper bound to stability such that when the bound is exceeded the flow is definitely unstable. On the other hand the energy method delineates the zone of definite stability with respect to finite amplitude disturbances. This zone may be further extended by a variational method as discussed by Joseph [3].

In the linearized theory it is possible to assume, the time dependency part of the disturbance in the exponential form i.e., in the form of $e^{-i\omega t}$, where the frequency ω is complex. If the imaginary part of ω is positive, the disturbance will increase with time. But actually this is not the case. As it amplifies it must eventually reach a size such that the mean transport of momentum by the finite fluctuations is appreciable, and such that the associated mean stress known as Reynolds stress has an appreciable effect on mean flow. Due to the appreciable effect of the Reynolds stress, the mean flow gets distorted. This distortion of the mean flow modifies the rate of transfer of energy from the mean flow to the disturbance and since this energy is the cause of the growth of the disturbance there is a modification of the growth of the latter (Stuart [4]). It is found that, in many cases, an equilibrium state may be possible in which the rate of transfer of energy from the (distorted) mean flow to the disturbance balances precisely the rate of viscous dissipation of the energy of disturbance. The equilibrium flow may exist in supercritical conditions if this Reynolds number R is above the critical value R_c corresponding to linearized theory. On the other hand subcritical instability may occur in a flow if this flow is unstable with respect to finite amplitude disturbances before it becomes unstable with respect to infinitesimal disturbances. It should be noted that if equilibrium flow is established, it does not mean that it is a stable flow at all Reynolds numbers above the critical value which is given by linearized theory. Further the equilibrium flow must become unstable in some way before the flow becomes turbulent.

Let us now examine the nature of the flow resulting from the absolute instability of the basic steady flow at large Reynolds numbers. We examine in particular what happens when the Reynolds number R exceeds R_c (the critical Reynolds number according to linearized theory) slightly. When the time dependence in the linear theory is of the form $e^{-i\omega t}$ with $\omega = \omega_1 + i\gamma_1$, it follows that when $R < R_c$, γ_1 is negative. When $R = R_c$, there is one frequency whose imaginary part is zero. While if R exceeds R_c slightly, γ_1 is positive but is much smaller than ω_1. In this case it is possible to assume the perturbed quantities in the form $A(t).\,f(x, y, z)$, where f is some complex function of co-ordinates, and the complex amplitude is

$$A(t) = const.\, e^{\gamma_1 t}.\, e^{-i\omega_1 t}. \qquad (1)$$

The equation (1) will be valid only when R is slightly greater than R_c i.e. only for a short interval after the break down of the steady flow.

Let us now calculate the modulus $|A|$ of the amplitude of the disturbance when R exceeds R_c slightly. For small values of t during which (1) is valid we may write $d|A|^2/dt = 2\gamma_1 |A|^2$, which is just equal to the first term of the expansion of $d|A|^2/dt$ in a series of powers of A and A^* (complex eonjugate of A). As the modulus of the amplitude $|A|$ increases (still remaining small), the subsequent terms in this expansion should be taken into account. The next terms in the expansion for $d|A|^2/dt$ would be third order in A. However we are not interested in the exact value of $d|A|^2/dt$ but only on its time average. This average is to be taken over a time large compared with the period $2\pi/\omega_1$ of the disturbance. Since $\omega_1 >> \gamma_1$, this period is much smaller than $1/\gamma_1$, which is a measure of the time