In a double sided short stator linear induction motor, the stator winding as well as the air gap are discontinuous at the entry end and the exit end. Because of these discontinuities, the conventional analysis of a rotary type induction motor cannot be directly applied to this motor. The end effects in the linear induction motor give rise to non-uniform flux distribution in the air gap and considerably reduce the net thrust developed.

Considerable amount of theoretical and experi*
mental study has been carried out by several investigators on the end effects in a linear induction motor (I.I.M) ${ }^{1-3,7-23,25,26}$. Notable among the theoretical analyses are those proposed by Yamamura ${ }^{1}$ and Nasar ${ }^{2}$. In the 1 -dimensional analysis given by Yamamura ${ }^{1}$ the effects of discontinuity in the winding as well as the stator iron are taken into account. But the boundary condition assumed at the entry end is somewhat approximate in the sense that it is valid for high velocities of the rotor. Nasar ${ }^{2}$ in his 1-dimensional analysis of the LIM has simulated the condition of finite length of the stator
iron by assuming fictitious reactive current sheets to be present beyond the active length of the stator. The 1-dimensional model assumed in these analyses is too much of an approximation, since it completely ignores the tangential component of the flux density in the air gap: In order to accommodate the normal as well as the tangential components of the air gap field, Nasar ${ }^{7}$ and Yamamura ${ }^{1}$ have also proposed 2-dimensional analyses. However to keep the problem amenable for a solution, they have assumed discontinuity only in the stator winding and not in the stator iron. Under these assumptions, the solution to the field problem can be obtained directly using Fourier-transform method. They have proposed numerical solutions for the case where both the discontinuity in the iron and the discontinuity in the winding are to be taken into account. Dukowicz ${ }^{3}$ in his analysis has considered the effects of finite winding length as well as finite iron length, but the solution given is applicable only for small air gap machines.

The analysis presented in this thesis leads to an analytical solution for the 2-dimensional air gap field of a linear induction motor taking into account the discontinuity in the stator iron. The solution to the field problem is obtained by the application of the

Wiener-Hopf technique and its extension ${ }^{4}$. If the exit end effect can be ignored at the entry end and vice-versa, it is shown that an exact solution can be obtained to the 2-dimensional air gap field. On the other hand, if both the entry and exist end effects are to be considered simultaneously, the solution can be obtained by using the so called approximate Wiener-Hopf technique. A special feature of this thesis is that a linear induction motor whose stator length is somewhat longer than the winding length is first analysed. The general solution thus obtained is made use of (i) to obtain the solution for a stator where the iron length is same as the winding length (ii) to obtain the solution of a machine with finite winding length but infinite iron length and (iii) to obtain the solution of a double layer staggered winding LIM. This type of approach clearly brings out the individual effects of the discontinuity in the winding and the discontinuity in the iron on the flux distribution and the thrust in a IIM.

The development of the thesis, chapterwise, is as follows :

In Chapter-I, the entry end effect alone is analysed assuming the stator to be long enough to

neglect the exit end influence at the entry end. The formulation of the problem gives rise to a 2-part mixed boundary value problem, which can be solved exactly by using the Wiener-Hopf technique. It is shown that the entry end effects are mainly due to the discontinuity in the winding, which prevents the rise of flux density to its normal value over considerable distance from the entry end. The effect is particularly high at small values of slip. The discontinuity in the winding at the entry end generally results in a loss of thrust. The discontinuity in the stator iron gives rise to a sharp rise of flux density at the tip of the entry end on the stator surface, but since this lasts over a very short distance, its effect on the thrust is not appreciable.

In Chapter-II, the exit end effects alone are analysed neglecting the influence of the entry end. It is shown that the exit end effects are mainly governed by the discontinuity in the iron. The discontinuity in the iron gives rise to a very large peak of flux density on the stator surface near the exit end and also to an additional driving thrust which can reach an appreciable value at small values of slip including zero slip.

In Chapter-III, the analysis is carried out by considering both the entry end and the exit end effects simultaneously. The formulation of the problem gives rise to a 3-part mixed boundary value problem, which can be solved by using the approximate Wiener-Hopf technique.

In Chapter-IV, the 2-dimensional analysis is extended to a staggered winding machine. Before applying the results of this analysis to a practical machine, it is necessary to suitably correct the air gap length and the rotor conductivity to account for the transverse end effects. These correction factors, derived from Bolton's ${ }^{10}$ analysis are also given. The theoretical results obtained are compared with the test results obtained by Coho et. al. ${ }^{18}$.

$$
\text { Finally, in Ghapter- } V \text {, a 1-dimensional }
$$

analysis' is presented with suitable correction factors which take into account the transverse end effects, the peripheral leakage flux in the air gap and the skin effect in the rotor sheet, to show that for all practical purposes, the results of the 1-dimensional model with these correction factors will approximately give the same results as the 2-dimensional analysis and will suffice for predicting the performance of a practical machine.

In the literature cited earlier, extensive test data is also available 1,17,18,20-22. The validity of the different analyses proposed in this thesis is checked by evaluating the performance of two motors of Yamamura ${ }^{1}$ and the motor of Coho et.al. ${ }^{18}$ in respect of air gap and yoke flux density distributions as well as thrust and normal forces. A close agreement between the test values and the theoretical results is obtained.

The salient points of the thesis are as follows :

1. An analytical solution is presented for the 2-dimensional field equations of a linear induction motor taking into account the discontinuity in the stator winding as well as stator iron.
2. The 2-dimensional analysis clearly brings out the nature of normal and tangential components of the flux density distribution on the stator surface as well as the rotor surface.
3. It is shown that the loss of thrust is mostly due to the discontinuity in the
winding at the entry end rather than due to discontinuity in the iron.
4. It is shown that the discontinuity in the iron at the exit end gives rise to a driving thrust, which could reach appreciable values at zero slip.
5. The theoretical results as predicted by the analysis are compared with test results in respect of air gap flux distribution as well as forces and are found to be quite satisfactory.

## LIST OF PRINCIPAL SYMBOLS

( NKS system of Units is used throughout)
$\begin{array}{ll}\Delta_{Z}(s, y, t) \quad & z-\text { Component of Vector potential }= \\ & \operatorname{Re}[A(x, y) \exp (j \omega t)] \\ A^{*}(s, y) \quad \text { Double sided Laplace Transform of } A(x, y)=\end{array}$

$$
\int_{-\infty}^{+\infty} A(x, y) \exp (-s x) d x
$$

$B_{x}, B_{y} \quad x$ - and $y$ - components of air gap Flux Density
$B_{c}$
Yoke Flux Density
2. Thickness of rotor sheet
e Induced voltage
$f_{1}(s, g) \quad$ Function of $s$ and $\varepsilon=$

$$
\mu_{0} \sigma d(v s+j \omega) \frac{\sin s g}{s}+\cos s g
$$

$f_{2}(s, g) \quad$ Function of $s$ and $\varepsilon=$

$$
\mu_{0} \sigma d(v s+j \omega) \sin s g-s \operatorname{cossg}
$$

$\mathrm{F}_{\mathrm{X}}, \mathrm{F}_{\mathrm{y}} \quad \mathrm{x}$ - and y - Components of thrust
$G_{+}(s)$

$$
\frac{\sum_{n=1}^{\infty}\left(1+\frac{s}{\delta_{n}}\right)}{\sum_{n=1}^{\infty}\left(1+\frac{s}{\beta_{n}}\right)}
$$

$\frac{n_{n=1}^{\infty}\left(1-\frac{s}{\alpha_{n}}\right)}{\prod_{n=1}^{\infty}\left(1-\frac{s}{y_{n}}\right)}$
Residue of $\frac{1}{G_{+}(s)}$ at $s=-\delta_{n}$
$G_{-}^{(R)}\left(y_{n}\right)$
Residue of $G_{-}(s)$ at $s=y_{n}$

Air gap length
h
$I_{1}$
Height of the stator block
RMS value of stator phase current
Peak value of linear current density
in the stator winding
z - component of linear current density in the stator. ( $\Lambda / \mathrm{m}$ )
$j_{2 x}, j_{2 z}$
$x$ - and $z$ - components of current
density in the rotor ( $\Lambda / \mathrm{m}^{2}$ )
k
$\pi / \tau$

| $k_{1}$ | Magnetic leakage Factor |
| :---: | :---: |
| $k_{g l}$ | Air gap correction factor due to peripheral leakage |
| $\mathrm{k}_{\mathrm{gt}}$ | Air gap correction factor due to transverse end effects |
| ${ }^{k} \sigma_{S}$ | Conductivity correction factor due to rotor skin effect |
| ${ }^{k}$ ot | Conductivity correction factor due to transverse end effects |
| I | Stator winding length |
| $\mathrm{N}, \mathrm{T}$ | Number of turns per phase |
| P | Number of poles, Power |
| S | Slip |
| S | Operator of double sided Iaplace Transform |
| V | Scalar potential, Voltage |
| v | Rotor velocity in $x$-direction, $\mathrm{v}_{5}(1.0-5)$ |
| $\mathrm{v}_{\mathrm{S}}$ | Synchronous speed |
| W, $2 W_{1}$ | Width of stator in z-direction |
| $2 W_{2}$ | Rotor width in z-direction |


| $\alpha_{n},-\beta_{n}$ | Zeros of $\mathrm{f}_{1}(\mathrm{~s}, \mathrm{~g})$ |
| :---: | :---: |
| $\gamma_{\mathrm{n}},-\delta_{\mathrm{n}}$ | Zeros of $\mathrm{f}_{2}(\mathrm{~s}, \mathrm{~g})=$ |
|  | $s^{2}-\nabla / \mu_{0} \sigma s-j \omega \mu_{0} \sigma$ |
| $\psi(s)$ | $\int_{0}^{L} \Lambda(x, g) \exp (-s x) d x$ |
| $\psi_{+}(s)$ | $\int_{0}^{\infty}(h(x, g) \exp (-s x) d x$ |
| $\psi_{\sim}(s)$ | $\int_{-\infty}^{0} A(x, g) \exp (-s x) d x$ |
| $\psi_{+}^{\prime}(\mathrm{s})$ | $\int_{0}^{\infty}\left[\frac{\partial \Lambda}{\partial y}\right]_{y=g} \exp (-s x) d x$ |
| $\psi_{-}^{\prime}(\mathrm{s})$ | $\int_{-\infty}^{0}\left[\frac{\partial A}{\partial y}\right]_{y=g} \exp (-s x) d x$ |
| $p$ | Resistivity of Rotor sheet |
| $\sigma$ | Conductivity of Rotor sheet |
| $\tau$ | Pole pitch |
| $\omega$ | Angular Velocity, $2 \pi f$ |

