INTRODUCTION AND SUMMARY

Investigation of the behaviour of nonlinear systems is a very interesting and important field of study, mainly because of the following reasons. Firstly, almost all physical systems possess nonlinearities and hence a study of the behaviour of such systems becomes imperative for a meaningful probe into the behaviour of most of the systems that we come across. Secondly, a very large array of interesting phenomena like limit cycles, jump resonance, subharmonic oscillations and asynchronous excitation etc., are exhibited by nonlinear systems. Lastly, one has the challenge of some very formidable problems associated with the analysis of such systems. These problems occur both at the conceptual and analytical level on the one hand, and at the level of the involved computational complexity on the other.

Ascertaining stability, investigating the limit cycle behaviour, analysis of forced harmonic response including multimodal behaviour and jump resonance [17,24], forced subharmonic oscillations [71,74], quenching of limit cycles by high frequency inputs -- deterministic and nondeterministic [11,21,46,48,49,50,64,66], asynchronous excitation [38,64] and investigation of the stability of forced oscillations [72] constitute problems of major interest in the analysis of nonlinear systems.

The Direct method of Liapunov [20], the Popov criterion [16,27,41,42,51] and other techniques making use of functional analysis [2,7,33,59] have been developed for investigating the absolute stability, and tne stability in the **small** of systems incorporating one or more nonlinear elements.

Even for systems incorporating a single nonlinear element, except in a few cases, exact analytical solutions are not possible. Consequently recourse has to be made to techniques that yield approximations to the system behaviour. A number of techniques, like phase plane analysis, the method of iteration, the method of perturbation, the averaging methods, the method of harmonic balance, the describing function (DF) method and the dual input describing functions (DIDF) have been developed [3, 19] to aid the analysis of nonlinear systems. However, while the application. of the approximate methods results in a marked reduction in the complexity of analysis for systems incorporating only one nonlinear element, the complexity for two and higher dimensional systems remains formidable even when these approximation techniques are employed.

The importance of system structure in the development and selection of the technique most suited to the analysis of a particular problem has been implicitely recognized in literature concerned with two and higher dimensional nonlinear systems. For example, Gelb [18] has investigated oscillatory behaviour of systems incorporating two or more nonlinear elements in a single loop. In these cases the assumed symmetry of the system has been exploited for reducing it to an equivalent system incorporating only one nonlinear element. Jud [28], Gran and Rimer [22], Davison and Constantinescu [15] have considered another structure incorporating a number of nonlinear elements each with associated linear elements in tandem, forming a number of parallel paths having a common feedback. Although systems having more general configurations have also been analysed [34,44,69], no systematic investigation of the various possible system configurations for nonlinear multivariable systems seems to have been made.

The literature pertaining to systems incorporating a single nonlinear element is well documented in a number of text books [3,9,20,39], review papers [4,5] and doctoral dissertations [64]. This brief review, therefore, confines attention to literature pertaining to systems incorporating two or more nonlinear elements.

Lindgren and Pinkos [32] have investigated the stability of symmetrical two dimensional nonlinear control systems by using Liapunov functions. However, in this analysis also symmetry considerations permit the problem to be effectively reduced to a single dimensional problem thus reducing the complexity of analysis.

The frequency domain stability criterion of Jury and Lee [29] that extends the absolute stability analysis of Popov to nonlinear multivariable systems has also been obtained by McGee [36] by establishing a condition for the existence of a Lure type Liapunov function. Moore and Anderson [42] have extended circle criterion to multivariable systems incorporating a finite number of time varying feedback elements. Huseyin [26] has extended this work to systems incorporating many nonlinear time varying feedback elements.

Byrne [12] has proposed a method for determining absolute stability for 2-channel symmetric nonlinear systems where the nonlinearities are restricted in sectors as well as in slopes.

A number of investigators [16,25,27,32,41,42,51] have contributed to the generalization of the Popov criterion to nonlinear multivariable systems.

McClamroch [34] considered a general state variable representation of a nonlinear multivariable system and obtained inequalities defining the sectors permissible for the nonlinear characteristics for ensuring zero input stability of the system.

Shankar [61] used Greshgorin's theorem on the bounds of eigenvalues to establish sufficient conditions for all the eigenvalues of a matrix P [$P = \underline{A}(j\omega) + \underline{A}^*(j\omega)$, where $\underline{A}(j\omega) = (I + j\omega Q)\ \underline{G}(j\omega) + K^{-1}$], to be positive for ensuring the positive definiteness of P. However, these procedures suffer from the trial and error involved in finding the best values of q_{ii} which determine the slopes of the Popov lines as well as radii of the banding circles.

Rosenbrock [58] and Cook [13] have derived results that are valid for nonlinearities confined to sectors. Both these results require that for stability the m-banded frequency response loci of the diagonal elements $\underline{G}_{ii}(j\omega)$ satisfy the single variable circle criterion with respect to circles with real axis diameter that are dependent on the sector bounds of the nonlinearities.

Rao and Baliga [56] and McClamroch and Lanculescu [35] have considered a specific type of interconnection of two nonlinear subsystems and developed a stability condition which depend explicitely on the properties of the subsystems and the interconnection parameters. The stability condition in the form of inequality expressions is a function of the product of interconnection coefficients related to the sector bounds of the nonlinearities.

Blight and McClamroch [10] have considered a certain class of multiloop systems and established a stability criterion which involves the individual Nyquist plots of the linear scalar subsystems and a certain positivity condition pertaining to the nonlinear subsystems. The derivation makes use of the functional analysis approach and depends on the hyper stability concept introduced by Popov [52].

Several other papers [2,7,33,59] have also investigated the absolute stability of nonlinear multivariable systems making use of functional analysis.

Gelb [18] and Lindgren [31] have investigated limit cycles in symmetric nonlinear systems by reducing the problem to an equivalent single loop system and basing the analysis on the describing function (DF) for the nonlinear elements.

Jud [28] has described a technique for investigating the limit cycle conditions for a general system configuration incorporating any number of parallel forward paths, each of which contains linear and nonlinear elements. The analysis employs the DF loop gain technique described by Schiring [60] wherein the input to each nonlinear element must be expressed in terms of the input to any one of the nonlinear elements. Thus, the loop gain equation formed in the frequency domain effectively involves two unknowns, namely ω and the amplitude of the input to the nonlinear element , which can be determined from the two equations obtained by separating the real and imaginary parts. A graphical solution is normally resorted to but the procedure becomes very cumbersome for systems incorporating a number of nonlinear elements.

Viswanadham and Deekshatulu [69] have analysed the self-oscillations in a two variable system by employing the DF approach in conjunction with the D-partition technique [62]. However the method is confined to only those nonlinearities whose DF expressions are explicit functions of input .

Barron [8] has developed a digital computer algorithm, the Harmonic Balance Algorithm (HBA), for analysing limit cycles in autonomous nonlinear mutlivariable systems. This algorithm makes use of the DF approach and accounts for the influence of several harmonic components.

Nugent and Kavanagh [45] and Rasis and Bonen [57] have analysed the self-oscillation in a nonlinear multivariable system incorporating a two dimensional relay characteristics by employing extensions of the Tsypkin approach [25]. Negescu and Sebastian [43] have developed a grapho-analytical method for determination of self-oscillation in systems incorporating two nonlinear memoryless, energyless elements separated by a linear device.

Using the DF approach Woon has developed a flexible criterion for the absence of limit cycles which appears to be less conservative in comparison with other criteria [73].

For two dimensional **systems** with uncoupled nonlinearities Ramani [3] showed that sufficient conditions for the non-existence of a limit cycle could be checked from plots of the DF loci for the two nonlinearities and the frequency response loci $\underline{G}_{11}(j\omega)$, $\underline{G}_{22}(j\omega)$ and the characteristic loci, $\underline{\lambda}(j\omega)$, of G(s). When these conditions are not satisfied, the parameters of a possible limit cycle can be evaluated from a complex equation developed from closed loop considerations by selecting a suitable complex constant.

Various theorems of matrix algebra have also been made use of for establishing sufficient conditions for the non-existence of limit cycle [37,55]. However these conditions ignore the relationships between the frequency and amplitude of the inputs to the nonlinearities that would exist for a limit cycle and these are often conservative [5].

Gray and Nakhla [23] analyse the limit cycles in multivariable nonlinear systems, by solving Harmonic Balance equations which consider the fundamental along with other superharmonic components by employing an iterative procedure. They have laid considerable emphasis on the computational aspects and also give a graphical interpretation in the frequency domain.

Kazakov [30] has employed a statistical linearization technique to analyse nonlinear multivariable control systems subjected to random disturbances.

Nikiforuk and Wintonyk [44] have investigated the response of two dimensional nonlinear systems subjected to sinusoidal inputs by employing a DF approach that involves a considerable amount of graphical manipulation .

Viswanadham et al [70] have given attention to the problem of design of symmetrical two dimensional nonlinear multivariable systems subjected to step inputs.

The present work is mainly concerned with the analysis of two dimensional nonlinear multivariable systems.

Two dimensional nonlinear multivariable systems constitute a very important class of systems. In many practical situations two servomechanisms are employed to control two distinct variables in a plant wherein the dynamics of the two variables is interlinked e.g. two axis control of positioning servos for radar antenna and machine tools [44], while in many situations the systems inherently possess dynamical characteristics that can be appropriately modelled as a two dimensional nonlinear multivariable system e.g. coupled core reactors etc. [51,54]. The analysis of self oscillations in systems belonging to this class is, therefore, a problem of considerable practical importance. Analysis of such systems is also important for the insight that is likely to be provided into the behaviour of higher dimensional nonlinear multi-variable systems and the choice of appropriate techniques for their analysis.

Specifically, this Thesis confines attention to the following aspects :

(a) Investigation into the nature of possible structures for two dimensional nonlinear multivariable systems.

(b) Systematic analysis of the self oscillations in such systems.

(c) Evolution of a method suitable for implementation on a digital computer and the development of computer algorithms.

(d) Signal stabilization of two dimensional nonlinear multivariable systems exhibiting self oscillations.

(e) Analysis of asynchronous excitation in two dimensional multivariable nonlinear systems.

The development of the Thesis is along the following lines :

Chapter 1 investigates the nature of possible system structures that a general two dimensional nonlinear multi-variable system can possess. Subsequently, several important subclasses of this general structure are obtained by successive simplifications. Next, necessary conditions namely (i) Phase, (ii) Gain, and (iii) Amplitude Ratio conditions that have to be satisfied for sustained self-oscillations in such systems are formulated. Several methods based on the DF analysis have been developed for a systematic analysis of self-oscillation in systems possessing structures of increasing complexity. A special feature of this chapter is the extension of the application of the Universal Chart [3,63,64] to the analysis of self-oscillations in two dimensional nonlinear multivariable systems. This leads to a systematic procedure for analysing possible self-oscillations with considerable economy in computational and graphical work. These methods lead to the determination of the frequency as well as the other parameters associated with the self-oscillations. All these methods are illustrated through application to specific examples and the results are compared with those obtained from digital simulation of the systems.

Chapter 2 is concerned with the modification and computerization of one of the more general methods developed in Chapter 1. Firstly the necessity for the modification leading to ease in computer implementation is examined and subsequently computer algorithms are developed for analysing self-oscillation in systems possessing the most general two dimensional nonlinear multivariable structure. The method is illustrated through application to specific examples and the results obtained from analysis are compared to those obtained from digital simulation.

Chapter 3 examines the feasibility of signal stabilization of two-dimensional multivariable nonlinear systems exhibiting self-oscillation by employing an auxiliary high frequency sinusoidal input. Methods for analysing the stabilization of nonlinear single variable systems

have been extended for application to systems possessing a two dimensional nonlinear multivariable structure. The interesting phenomena of desynchronization and synchronization are also analysed and the results of the analysis are compared with results obtained from digital simulation. Finally, the possibility of a two dimensional nonlinear multivariable system exhibiting asynchronous excitation in the presence of a high frequency sinusoidal input has also been examined leading to an analysis of this interesting phenomenon. The results obtained from the analysis are again compared with those obtained from digital simulation for a specific system.